4.2 Bottleneck accumulation of hybrid bosons in a ferrimagnet

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Our studies of the magnon Bose-Einstein condensate (BEC) in a film of yttrium iron garnet ($Y_3Fe_5O_{12}$, YIG) resulted in the discovery of a novel condensation phenomenon mediated by magneto-elastic interaction: A spontaneous accumulation of hybrid magneto-elastic bosonic quasiparticles at the intersection of the lowest spin-wave mode and a transversal acoustic wave.

We studied a magnon gas populated by parametric microwave pumping in an in-plane magnetized 6.7 µm thick YIG film by wavevector-resolved Brillouin light scattering spectroscopy (BLS) [1]. The pumping circuit (see Fig. 1a) is fed with 1.5 µs long microwave pulses with a carrier frequency $f_p = 13.62$ GHz. A microstrip resonator with a width of 0.5 mm creates an alternating Oersted field along the direction of a bias magnetic field, realizing, thus, parallel parametric pumping [2].

BLS intensity maps showing the population of the pumped magnon spectrum (an example is shown in Fig. 1b as a function of frequency and wavenumber are presented in Fig. 2a-b for two different bias magnetic fields and a relatively small pumping power of 2.6 W. In spite of the fact that the threshold of magnon BEC formation is still not reached at such power levels one can see a strong and sharp population peak, which appears in the region of the hybridization between the magnon and the transversal acoustic phonon dispersion branches (see white lines in Fig. 2). When the bias magnetic field is shifted from 1735 Oe to 1265 Oe, the population peak shifts in frequency and wavenumber together with the magneto-elastic crossover region. At the same time, there are no peculiarities in the thermal spectrum, measured at the same conditions, but without application of pumping (see Fig. 2c). It means that the observed phenomenon is associated with the accu-

![Fig. 1: a) Experimental setup. The probing laser beam is focused onto a spot with 25 µm diameter on a YIG film placed on top of a microstrip resonator. The light inelastically scattered by magnons is redirected to a Fabry-Pérot interferometer for frequency and intensity analysis. Wavenumber-selective probing of magnons with wavevectors $\pm \mathbf{q} \parallel \mathbf{H}$ is realized by varying the angle of light incidence $\Theta_{q \parallel H}$. b) Magnon-phonon spectrum of a 6.7 µm-thick YIG film for $H = 1735$ Oe. 47 thickness modes with $\mathbf{q} \parallel \mathbf{H}$ are presented. The upper thick curve shows the lowest mode with $\mathbf{q} \perp \mathbf{H}$. The arrow illustrates the magnon injection process.](image)
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\( \omega \) parametrically exited magnons redistribute over the entire \( \mathbf{q} \)-space such that the magnon number \( n(\mathbf{q}) \) moves toward lower frequencies. At relatively high frequencies the main contribution to the scattering amplitude \( T_{\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4} \) is given by the exchange interaction for which \( T_q \equiv T_{\mathbf{q},\mathbf{q},\mathbf{q},\mathbf{q}} \propto q^2 \).

Therefore, the most efficiently thermalized magnons have the largest wavenumbers \( q \) and, thus, for the case of an in-plane magnetized YIG film, belong to the lowest magnon mode with \( \mathbf{q} \parallel \mathbf{H} \) [3, 4].

In the presence of magnon-phonon coupling the unperturbed magnon \( \omega^m_q \) and phonon \( \omega^p_q \) spectra, shown in Fig. 3a by dashed lines, split up in the Upper and Lower Magneto-Elastic Modes (UMEM, \( \Omega^U_q \) and L-MEM, \( \Omega^L_q \)), shown in Fig. 3a by solid lines. The population dynamics of these modes is determined by both intramodal (2L=2L, 2U=2U) and intermodal (2L=2U) scattering of hybrid magneto-elastic bosons with the following conservation laws and interaction amplitudes:

\[
\begin{align*}
\Omega^L_{\mathbf{q}_1} + \Omega^L_{\mathbf{q}_2} &= \Omega^L_{\mathbf{q}_3} + \Omega^L_{\mathbf{q}_4}, \quad T^L_{\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4}, \\
\Omega^U_{\mathbf{q}_1} + \Omega^U_{\mathbf{q}_2} &= \Omega^U_{\mathbf{q}_3} + \Omega^U_{\mathbf{q}_4}, \quad T^U_{\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4}, \\
\Omega^L_{\mathbf{q}_1} + \Omega^L_{\mathbf{q}_2} &= \Omega^L_{\mathbf{q}_3} + \Omega^L_{\mathbf{q}_4}, \quad T^L_{\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4}.
\end{align*}
\]

All interaction amplitudes \( T_{\cdots} \) originate from the four-magnon interaction amplitude \( T_{\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4} \) and, thus, are proportional to the magnon contribution to MEMs. It can be shown that this contribution is given by \( \cos \varphi_q \) regarding the L-MEM and \( \sin \varphi_q \) regarding the U-MEM, whose values are determined by the dimensionless frequency distance from the crossover \( o_q \):

\[
\cos \varphi_q = \frac{1}{2}(1 + o_q(\alpha_q^2 + 1)^{-1/2}), \quad o_q = (\omega^p_q - \omega^m_q)/\Delta,
\]

where \( \Delta = \Omega^U_q - \Omega^L_q \) is the magneto-elastic frequency and \( \mathbf{q}_0 \) is the crossover wavevector at which \( \omega^p_{\mathbf{q}_0} = \omega^m_{\mathbf{q}_0} = \omega_0 \).

In particular

\[
T^L_q = T_0 \cos^4 \varphi_q, \quad T^U_q = T_q \cos^2 \varphi_q \sin^2 \varphi_q.
\]
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Fig. 3: Theory. a) Calculated magnon-phonon spectrum near the hybridization region. b) Interaction amplitudes $T_{LL}^{UU}$, $T_{LL}^T$ and $T_{UU}^T$, normalized by $T_0$, vs dimensionless distance from the hybridization crossover $\alpha_q$. c)-e) Schematic representation of the scattering and cross-scattering 4-particle processes in the hybridization region. f) Dimensionless L-MEM density $N_{LL}^{L}$ for different values of parameter $\alpha$.

and $T_{UU}^{UU} \simeq T_q \sin^4 \phi_q$, where $T_q \equiv T_{UU}^{UU} / T_0$. Plots of $T_{LL}^{LL}$, $T_q^{LU}$ and $T_{UU}^{UU}$ vs. $\alpha_q$ are shown in Fig. 3b.

As expected, for positive $\alpha$ (when $q > q_0$), the L-MEM becomes a pure magnon mode and $T_{LL}^{UU} \rightarrow T_q$, while $T_{UU}^{UU} \rightarrow 0$ because the U-MEM becomes a pure phonon mode (see Figs. 3a and 3b). The cross-amplitude $T_{LU}^{UU}$ requires the presence of magnon parts both in the L- and U-MEMs, therefore its value is significant only in the hybridization region, where $|\alpha| \lesssim 1$, as shown by the solid line in Fig. 3b. As it is clear from Fig. 3b, for $\alpha > 1$ we can take into account only the intramodal $2L \Rightarrow 2U$ scattering described by Eq. (1c), which conserves the total number $\int N_{LL}^{L} \, dq$ of L-MEM quasi-particles. This allows us to write the balance equation for $N_{LL}^{L}$ in the form of the continuity equation

$$\partial N_{LL}^{L} / \partial t + \partial \mu_q^{L} / \partial q = 0,$$

where $\mu_q$ is the L-MEM flux towards the hybridisation area. (4)

Using a classical Hamiltonian approach we estimated $\mu_q$ in the vicinity of the hybridization as

$$\mu_q^{L} \simeq q_0^3 (T_{LL}^{UU})^2 (N_{LL}^{L})^3 / \omega_q.$$

(5)

In the steady-state Eq. (4), which neglects the intermodal $2L \Rightarrow 2U$ scattering described by Eq. (1c), the flux $\mu_q^{L}$ should be $q$ independent. Together with Eq. (3) this gives

$$N_{LL}^{L} \propto (T_{LL}^{UU})^{-2/3} \propto 1 / (\cos^{8/3} \phi_q) = \left[ (1 + \alpha_q (\alpha_q^2 + 1)^{-1/2}) / 2 \right]^{-4/3},$$

i.e. a fast population increase, as shown in Fig. 3f by curve 1. We call this effect “bottleneck accumulations of magneto-elastic bosons in the hybridization region”.

When $\omega_q$ goes down below $\omega_q$, the magnon part in the L-MEM goes to zero and $N_{LL}^{L} \rightarrow \infty$. However, the direct particle transfer from lower to higher MEMs makes $N_{LL}^{L}$ finite. This effect is proportional to $|T_{UU}^{UU}|^2$, shown in Fig. 3b by the solid line. In the presence of this process the steady-state balance equation (4) takes the form:

$$d \mu_q^{L} / dq = T_{UU}^{LU},$$

(7)
where $F_{LU}^{q}$ is the transport rate $N_{L}^{q} \to N_{U}^{q}$ caused by the intermodal $2L \Rightarrow 2U$ scattering. The $F_{LU}^{q}$ can be estimated as

$$F_{LU}^{q} \simeq q^{2} (T_{LU}^{q})^{2} (N_{L}^{q})^{2} N_{-}^{q} / \omega_{q},$$  \hspace{1cm} (8)$$

where $N_{-}^{q}$ is the density of U-MEMs at a sufficiently large negative $\omega$, say for $\omega \simeq -5$. Using estimates (3), (5) and (8) the balance Eq. (7) can be presented as:

$$\frac{d}{dq} \left[ \cos^{2} q_{p} (\mathcal{N}_{L}^{q})^{3} \right] = 3a (\mathcal{N}_{L}^{q})^{2} \cos^{4} q_{p} \sin^{4} q_{p},$$  \hspace{1cm} (9)$$

Here, $\mathcal{N}_{L}^{q} = N_{L}^{q} / N_{L}^{q+}$ is the dimensionless L-MEM density, normalized by the density $N_{L}^{q+}$ at a sufficiently large positive $\omega$, say at $\omega \simeq +5$. The dimensionless damping parameter $a = b N_{-}^{q} / N_{L}^{q+}$ contains the dimensionless coefficient $b$ (presumably of the order of unity) comprising numerical parameters that were not controlled in the estimates (5) and (8). The ordinary differential equation (9) can be solved in quadratures with the boundary conditions $\lim_{q \to \infty} N_{L}^{q} = 1$, giving the relative L-MEM population in the hybridization region

$$\mathcal{N}_{L}^{q} = \frac{1}{\cos^{8/3} q_{p}} \left[ 1 - a \int_{q}^{\infty} \frac{\sin^{4} q_{p} dp}{\cos^{4/3} q_{p}} \right].$$  \hspace{1cm} (10)$$

Solutions of Eq. (10) for different values of $a$ are shown in Fig. 3f. For $a = 0$ we return to Eq. (6) which corresponds to the maximum possible (asymptotic) bottleneck accumulation of the L-MEM density in the hybridization region giving an infinite growth of $\mathcal{N}_{L}^{q} \propto |q|^{8/3}$ for large negative $\omega$. For finite $a$ this growth is suppressed. However, when the transfer of quasi-particles from the L-MEM to the U-MEM is small ($a \ll 1$), the L-MEM density accumulation is quite significant. For large $a \simeq 1$ (see the two lower lines in Fig. 3f for $a = 1$ and 0.5), there is practically no MEM accumulation. It is necessary to note that the increase in the population of the U-MEM in the course of the BEC formation must lead to the suppression of the bottleneck effect and to the consequent saturation of the MEM population peak. This effect is clearly visible in our previous observations [5].

Concluding, our model describes the formation of the experimentally observed peak in the MEM population and, thus, evidences that this accumulation appears due to the bottleneck effect in the transfer of magneto-elastic bosons from high to low frequencies through the hybridization region. Financial support from the Deutsche Forschungsgemeinschaft (project INST 161/544-3 within the SFB/TR 49) and from EU-FET (Grant InSpin 612759) is gratefully acknowledged. D.A.B. is supported by a fellowship of the Graduate School Material Sciences in Mainz (MAINZ).

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