4.1 Magnetoelastic modes and lifetime of magnons in thin yttrium iron garnet films

D.A. Bozhko, A.A. Serga, and B. Hillebrands

In collaboration with A. Rückriegel and P. Kopietz, Institut für Theoretische Physik, Universität Frankfurt, 60438 Frankfurt, Germany

The spin-lattice interactions in magnetic insulators can often be ignored. In some cases, however, the coupling between the spin degrees of freedom and the lattice vibrations (phonons) plays a crucial role. For example, in ultrasound experiments one uses the spin-lattice coupling to study the properties of the spin degrees of freedom from the measurement of the propagation and the attenuation of sound waves [2]. The theory of magneto-elastic effects in magnetic insulators has been developed more than half a century ago by Abrahams and Kittel [3, 4], and by Kaganov and Tsukernik [5]. While in the past decades a few theoretical studies of magneto-elastic effects have appeared [6–11], recent experimental progress in the field of spintronics has revived the interest in the interactions between spin and lattice degrees of freedom [12]. In this work we calculate the effects of the spin-lattice coupling on the magnon spectrum of thin ferromagnetic films consisting of the magnetic insulator yttrium-iron garnet and present experimental results for the magnon damping in the dipolar regime, which have been obtained by means of time- and wave-vector-resolved Brillouin light scattering (BLS) spectroscopy [14].

It is generally established that the magnetic properties of YIG at room temperature can be obtained from the following effective quantum spin Hamiltonian [15–17]

\[
\mathcal{H} = -\frac{1}{2} \sum_{ij} \sum_{\alpha\beta} (J_{ij} \delta_{\alpha\beta} + D_{ij}^{\alpha\beta}) S_i^\alpha S_j^\beta - h \sum_i S_i^z ,
\]

where the spin operators \( S_i = S(R_i) \) are localized at the sites \( R_i \) of a cubic lattice with lattice spacing \( a \approx 1.2376 \text{ nm} \), the exchange couplings \( J_{ij} = J(R_i - R_j) \) connect the spins at nearest neighbour sites \( R_i \) and \( R_j \), and \( h = \mu H \) is the Zeemann energy due to an external magnetic field \( H \) along the \( z \) axis (where \( \mu = 2\mu_B \), and \( \mu_B \) is the Bohr magneton). The dipole-dipole interaction is

\[
D_{ij}^{\alpha\beta} = (1 - \delta_{ij}) \frac{\mu^2}{|R_{ij}|^3} \left( 3 \hat{R}_{ij}^\alpha \hat{R}_{ij}^\beta - \delta_{\alpha\beta} \right) ,
\]

Fig. 1: Thin YIG stripe of thickness \( d \) in a magnetic field \( \mathbf{H} = H \mathbf{e}_z \) along the direction \( \mathbf{e}_z \) of the long axis. We consider magnons with wave vector \( \mathbf{k} = |\mathbf{k}| \cos \theta_0 \mathbf{e}_z + |\mathbf{k}| \sin \theta_0 \mathbf{e}_y \) in the stripe-plane.

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where $R_{ij} = R_i - R_j$ and $\hat{R}_{ij} = R_i/R_{ij}$. Within the framework of the usual expansion in inverse powers of the spin $S$ the low-energy magnon spectrum of YIG can be quantitatively described if one chooses $J \approx 3.19\text{K}$ and $S = M_s a^3/\mu \approx 14.2$, where the saturation magnetization of YIG is given by $4\pi M_s = 1750\text{G}$. Due to the large value of the effective spin $S$ we may use the Holstein-Primakoff transformation [18] to express the spin operators in terms of canonical boson operators $b_i$ and $b_i^\dagger$. To describe a thin stripe we can work with an effective two-dimensional model, as explained in Ref. [17]. We consider geometry shown in Fig. 1.

One source of the spin-phonon coupling is the dependence of the true positions $r_i = R_i + X_i$ of the spins on the phonon displacements $X_i = X_i(R_i)$ at the lattice sites $R_i$. The resulting magnon phonon interaction can be derived from the effective spin-model (1) by expanding the exchange couplings $J_{ij} = J(R_i - R_j + X_i - X_j)$ in powers of the phonon displacements [19]. However, in collinear magnets such a procedure does not take into account the dominant source of the magnon-phonon interaction, which is generated by relativistic effects such as dipole-dipole interactions and spin-orbit coupling [10]. These effects involve the charge degrees of freedom so that they cannot be simply included in our effective spin model (1). To derive the proper quantized interaction between magnons and phonons in YIG, we therefore follow the semi-phenomenological approach pioneered by Abrahams and Kittel [3], which relies on the quantization of the phenomenological expression for the classical magneto-elastic energy [1,3–5,8,10]:

$$E_{me}[M, X] = \frac{n}{M_s^2} \int d^3r \sum_{\alpha\beta} \left[ B_{\alpha\beta} M_\alpha(r) M_\beta(r) + B'_{\alpha\beta} \frac{\partial M_\alpha(r)}{\partial r_\alpha} \cdot \frac{\partial M_\beta(r)}{\partial r_\beta} \right] X_{\alpha\beta}(r) , \quad (3)$$

After quantization of magneto-elastic energy it becomes possible to calculate the energy dispersion of the magneto-elastic modes (for details, see Ref. [1]). The energy dispersion of these modes is shown graphically in Fig. 2 for $k = k_e$ parallel to the in-plane magnetic field. Note that for this propagation direction the longitudinal phonon does not hybridise with the magnon dispersion because for $k_y = 0$ (corresponding to $\theta_k = 0$) the relevant hybridisation function [1] vanishes.

The Brillouin light scattering intensity is proportional to the transverse spin structure factor [21,22].

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**Fig. 2:** a) Dispersions of the magneto-elastic modes of a thin YIG stripe with thickness $d = 6.7\mu\text{m}$ in an external magnetic field $H = 1710\text{Oe}$, for $k = k_e$ parallel to the in-plane magnetic field; b) A magnified view of the hybridisation at the crossing of magnon and transverse phonon dispersions.
for the magnon damping on resonance at high temperatures, we carry out the frequency sum we obtain [1]

$$S_{\perp}(k, \omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i\omega t} \langle S^z_{-k}(0) S^z_k(t) + S^x_{-k}(0) S^x_k(t) \rangle.$$ \hspace{1cm} (4)

The Fourier transform of the spin-operator is defined as $M(k) \to g\mu_B \sqrt{N}S_k = g\mu_B \sum_i e^{-ik\cdot R_i} S_i$, where $n = N/V = 1/a^3$ is the unit cell number density.

An intensity plot of the transverse dynamic structure factor is shown in Fig. 3.

The damping $\gamma(k)$ of magnons with wave vector $k$ and energy $E_k$ can be obtained from the imaginary part of the self-energy $\Sigma(k) = \Sigma(k, i\omega)$ after analytic continuation to the real frequency axis,

$$\gamma(k) = -\text{Im} \Sigma(k, i\omega \to E_k + i\eta).$$ \hspace{1cm} (5)

After carrying out the frequency sum we obtain [1]

$$\Sigma_2(k, i\omega) = \frac{1}{N} \sum_{k', \lambda} \left\{ \frac{|\Gamma^\beta_{k,k',\lambda}|^2}{2m_\omega_{k-k'\lambda}} \left[ \frac{b(\omega_{k-k'\lambda}) - b(E_{k'})}{i\omega + \omega_{k-k'\lambda} - E_{k'}} + \frac{1 + b(\omega_{k-k'\lambda}) + b(E_{k'})}{i\omega - \omega_{k-k'\lambda} - E_{k'}} \right] 
- \frac{|\Gamma^\beta_{k,k',\lambda}|^2}{2m_\omega_{k+k'\lambda}} \left[ \frac{1 + b(\omega_{k+k'\lambda}) + b(E_{k'})}{i\omega + \omega_{k+k'\lambda} + E_{k'}} + \frac{b(\omega_{k+k'\lambda}) - b(E_{k'})}{i\omega - \omega_{k+k'\lambda} + E_{k'}} \right] \right\}.$$ \hspace{1cm} (6)

Here $b(\omega) = 1/(e^{\omega/T} - 1)$ is the Bose function. Since the experiments of interest to us are performed at room temperature which is large compared with all other energy scales, we may use the high temperature expansion of the Bose functions, $b(\omega) \approx T/\omega$. Setting now $\omega = E_k$ we obtain for the magnon damping on resonance at high temperatures,

$$\gamma_2(k) = \frac{\pi T E_k}{2mN} \sum_{k', \lambda} \left\{ \frac{|\Gamma^\beta_{k,k',\lambda}|^2}{E_k \omega_{k-k'\lambda}} \left[ \delta(E_k - E_{k'} + \omega_{k-k'\lambda}) + \delta(E_k - E_{k'} - \omega_{k-k'\lambda}) \right] 
+ \frac{|\Gamma^\beta_{k,k',\lambda}|^2}{E_k \omega_{k+k'\lambda}} \delta(E_k + E_{k'} - \omega_{k+k'\lambda}) \right\} = \gamma_2^{\text{he}}(k) + \gamma_2^{\text{he}}(k) + \gamma_2^{\text{con}}(k),$$ \hspace{1cm} (7-a)
compared with the phonon velocities, while at very small wave vectors in the regime around the minima of the dispersion, the velocity of the magnons is dominated by the conduction energy $\gamma_{\text{con}}(k)$.

In the long-wavelength regime where the exchange energy $\rho, k^2$ is much less than the characteristic dipolar energy $\Delta$ (i.e. $|k| \lesssim \sqrt{\Delta/\rho}$), the behavior of the magnon damping (7-a) strongly depends on the size $v(k) = |v(k)|$ of the group velocity $v(k) = \nabla_k E_k$ of the magnons in comparison with the phonon velocities. In the regime around the minima of the dispersion, the velocity $v(k)$ is small compared with the phonon velocities, while at very small wave vectors $v(k) > c_\lambda$. In this regime around the minima of the dispersion the decay rate of the magnons is dominated by the confluent process given in Eq. (7-d) because the Cherenkov processes are kinematically suppressed. In fact, in a substantial regime around the dispersion minima the quasi-particle velocity is small compared with the phonon velocities, so that we may approximate $E_{-k+q} \approx E_k - v(k) \cdot q$ and expand the decay rate in powers of $v(k)/c_\lambda$. The momentum integration in Eq. (7-d) can then be carried out and we obtain the confluent contribution to the high-temperature damping rate in the dipolar regime which is shown graphically as the thin dotted line in Fig. 4.

\[ \gamma_{\text{Che}}^{2a}(k) = \frac{\pi T E_k}{2mN} \sum_{q, \lambda} \frac{|\Gamma_{k,k+q,\lambda}^{\beta\beta}|^2}{E_{k+q,\lambda}^2} \delta(E_k - E_{k+q} + \omega_{q \lambda}), \]  

\[ \gamma_{\text{Che}}^{2b}(k) = \frac{\pi T E_k}{2mN} \sum_{q, \lambda} \frac{|\Gamma_{k,k-q,\lambda}^{\beta\beta}|^2}{E_{k-q,\lambda}^2} \delta(E_k - E_{k-q} - \omega_{q \lambda}), \]  

\[ \gamma_{\text{con}}^{2}(k) = \frac{\pi T E_k}{2mN} \sum_{q, \lambda} \frac{|\Gamma_{k-k+q,\lambda}^{\beta\beta}|^2}{E_{-k+q,\lambda}^2} \delta(E_k + E_{-k+q} - \omega_{q \lambda}). \]  

The contributions $\gamma_{\text{Che}}^{2a}(k)$ and $\gamma_{\text{Che}}^{2b}(k)$ are due to the Cherenkov type process where a magnon with energy $E_k$ emits or absorbs a phonon with energy $\omega_q$ and decays into a magnon with energy $E_{k\pm q}$. The last contribution $\gamma_{\text{con}}^{2}(k)$ describes a confluent scattering process where two magnons with energies $E_k$ and $E_{-k+q}$ decay into a phonon with energy $\omega_q$.

Fig. 4: Numerical evaluation of our result (7-a) for the damping rate of magnons in a thin YIG stripe at temperature $T = 300$ K, in the dipolar momentum regime. The plot is for a thin stripe of thickness $d = 6.7$ $\mu$m in an external magnetic field $H = 1710$ Oe, for $k = k \mathbf{e}_z$ parallel to the in-plane magnetic field. Solid lines correspond to the total damping rate, while the dashed and the dotted lines are the contribution from the Cherenkov and the confluent processes, respectively. The corresponding thin dotted line is the approximation in the dipolar momentum regime.
Fig. 5: Experimentally determined dependence of the total magnon damping $\gamma_{\text{tot}}$ on the in-plane wave vector in the dipolar regime where the magnon dispersion is dominated by the competition between the dipole-dipole and the exchange interaction. The shaded regions represent the estimated experimental uncertainties.

For a comparison of our calculation with experiments one should keep in mind that we have only considered the contribution from the magnon-phonon interactions on the damping of the magnons. Of course, in the real system there are other sources leading to magnon decay, such as magnon-magnon interactions or the elastic scattering of magnons by impurities. We therefore expect that the magnon damping due to magnon-phonon interactions is a lower limit to the experimentally observed magnon damping rate. In fact, our experimental data presented below show that in the dipolar regime magnon-phonon interactions are not the dominant source of magnon damping.

In order to determine the relaxation time of different groups of magnons, a measurement of the spectral distribution of magnon gas densities as a function of the frequency and wave vector using time- and wave-vector-resolved BLS spectroscopy [14] has been performed. Due to technical reasons only in-plane wave vectors from $0$ up to the $k_{\text{max}} = 11 \times 10^4 \text{cm}^{-1}$ are accessible. The measurements were performed using a YIG film with thickness $6.7 \mu m$, which was liquid-phase epitaxially grown on a $500 \mu m$ thick gallium gadolinium garnet substrate.

The magnon spectrum was populated by intensive thermalization [20, 23, 24] of magnons, which were injected using the parallel parametric pumping technique [25] at half of the pumping frequency $f_p = 13.62 \text{GHz}$. The bias magnetic field of $H = 1710 \text{Oe}$ was tuned to provide the excitation of parametric magnons at the ferromagnetic resonance frequency. In this case the direct transition of magnons to the bottom of the spin wave spectrum is prohibited by conservation laws. This ensures high efficiency of multi-stage four-magnon scattering which is necessary for thermalization and thus population of the spectrum.

We have measured the redistribution of thermalized magnons along the fundamental backward-volume magnetostatic spin wave mode as a function of time and wave vector. After switching off the pumping, the magnons are allowed to relax freely. By fitting the relaxation times for different groups of thermalized magnons we were able to extract the damping rates. The obtained dependence of the measured total damping rate $\gamma_{\text{tot}}$ on the in-plane wave vector for dipolar-exchange spin waves is shown in Fig. 5. Obviously, the value of relaxation rate is roughly three orders of magnitude larger than our calculated relaxation rate due to magnon-phonon interactions shown in Fig. 4. We thus conclude that in the long-wavelength dipolar regime other relaxation channels (in particular two-magnon scattering processes [26–29]) dominate the magnon damping. The rather irregular behavior of the measured damping rate in Fig. 5 suggests that elastic scattering of magnons by impurities might play an important role in this regime. Note that within the tolerance limits of the experiment the measured relaxation rate has a rather weak dependence on the in-plane wave...
vectors in the entire accessible range of wave vectors. In this respect the experimental results agree with our prediction of a momentum-independent damping rate in this regime.

Unfortunately, microscopic calculations of the magnon decay rates at room temperature, taking magnon-impurity and magnon-magnon interactions into account, are not available in the momentum range relevant for our experiment. One should keep in mind, however, that magnon-impurity scattering can only explain the momentum-relaxation of the magnon gas; for the equilibration of the different temperatures of the magnon and the phonon systems magnon-phonon interactions are essential.

For wave vectors in the regime where the exchange energy $\rho_s k^2$ exceeds the characteristic dipolar energy $\Delta$ we may ignore the dipole-dipole interactions in the magnon dispersion and approximate the long-wavelength magnon dispersion by $E_k \approx \hbar + \rho_s k^2$. Then the evaluation of the integrals in Eqs. (7-b)–(7-d) simplifies. We obtain for the Cherenkov part

$$\gamma_2^{\text{Che}}(k) \equiv \gamma_2^{\text{Che}}(k) + \gamma_2^{\text{Che}}(k) = \frac{TE_k m_s}{2} \sum_{\lambda} \frac{a^2}{c_\lambda^2} \times \int_0^{2\pi} \frac{d\varphi}{2\pi E_k} \frac{U_\lambda^2(\tilde{q}_\varphi)}{c_\lambda - v(k) \cos \varphi}.$$

(8)

where $v(k) = |k|/m_s$, the mass $m_s$ is defined via $\rho_s = 1/(2m_s)$.

A graph of the Cherenkov and confluent contribution to the high-temperature damping rate in the exchange regime is shown as the thin dashed line and thin dotted line in Fig. 6 respectively. As one can see, the magnon damping is strongly $k$-dependent. In particular, it exhibits peaks at the crossing points of magnon and phonon dispersions as well as velocities. It also increases by two orders of magnitude between dipolar and exchange regimes, which are dominated by confluence and Cherenkov processes, respectively. For the magnon lifetime $\tau(k) = 1/(2\pi\gamma(k))$, this implies values of the order of $50\mu s$ in the dipolar momentum range, while it can be as low as $480\text{ns}$ for exchange momenta.

In this work we have studied magneto-elastic interactions in experimentally relevant thin films of the magnetic insulator YIG. As the dominant sources of magneto-elastic interactions are due to relativistic effects [10] which cannot be taken into account within an effective model containing only spin degrees of freedom, we have used a semi-phenomenological approach [3], which relies on the quantization of a suitable phenomenological expression for the magneto-elastic energy. For the quantized theory we have then carefully derived the momentum dependence of the magneto-elastic interaction vertices within the framework of the conventional $1/S$-expansion for ordered
quantum spin systems. Using these vertices, we have explicitly calculated the leading contributions to the hybridisation between magnon and phonon modes, as well as the damping of the magnons due to spin-lattice coupling. The hybridisation has been shown to give rise to a characteristic minimum in the spin dynamic structure factor at the crossing point of magnon and transversal phonon dispersions, where the spectral weight is transferred from the magnons to the transverse phonon mode. The position of this minimum quantitatively agrees with the recent experimental observation of the magneto-elastic mode [30].

The damping at room temperature has been shown to be strongly momentum dependent. In the long-wavelength dipolar regime it is rather flat and almost exclusively driven by confluent magnon-phonon scattering processes where two magnons decay into a phonon or vice versa. In this regime, we have also presented new experimental results for the magnon damping obtained by wave-vector-resolved Brillouin light scattering spectroscopy. The fact that the experimental results for the magnon damping are roughly three orders of magnitude larger than our theoretical results indicate that in the dipolar regime magnon-phonon interactions are not the dominant source of magnon damping in our samples at room temperature. We suspect that in this regime the magnon damping is dominated by elastic scattering of magnons from impurities. On the other hand, in the short-wavelength exchange regime the damping is due to magnon-phonon scattering processes of the Cherenkov type and is two orders of magnitude larger than in the dipolar regime. The damping rate exhibits pronounced peaks at the crossing points of magnon and phonon dispersions and velocities. This agrees very well with the conclusions of the experiment [13], where the authors suggested that the spin-lattice relaxation in the dipolar regime should be much slower than in the exchange regime in order to reconcile their results with earlier work on the spin Seebeck effect.

The present work can be extended in two directions: on the theoretical side, it would be useful to have quantitatively accurate calculations of the magnon damping due to magnon-impurity and magnon-magnon interactions in the dipolar regime; we expect that this can provide a better explanation for our experimental results shown in Fig. 5, which is three orders of magnitude larger than the damping due to magnon-phonon interactions in this regime. Note, however, that recently Chernyshev [31] has considered spontaneous magnon decays of the $k = 0$ magnon in YIG due to magnon-magnon interactions in high magnetic fields. On the experimental side, it would be useful to measure magnon damping in the exchange regime and compare the data with our theoretical prediction shown in Fig. 6.

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References


