Fano profiles in transmission spectra of terahertz radiation through one-dimensional periodic metallic structures

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(Received 20 October 2010; accepted 19 November 2010; published online 15 December 2010)

The transmission properties of broadband terahertz radiation through one-dimensional periodic metallic structures are investigated experimentally, and the obtained results are explained theoretically. Characteristic minima known as Wood’s anomalies are observed in the transmission spectra. The measured line shapes, the so-called Fano profiles, are caused by the coupling between resonant surface plasmons excited on the metallic grating and nonresonant diffraction orders. Numerical simulations using classical electrodynamics are in good agreement with the experiments. © 2010 American Institute of Physics. [doi:10.1063/1.3526756]

Already in 1902, Wood observed anomalies in the reflected spectra of ruled metallic gratings using a white light source. These were later identified to be caused by surface plasmon polaritons (SPPs). Since then, many experiments have been carried out in order to fully understand and utilize the phenomenon. Transmission properties of electromagnetic radiation through periodic metallic structures, such as hole arrays and deep metallic gratings, were studied (see, e.g., Refs. 2–4). The terahertz range of the electromagnetic spectrum is well suited for investigating these effects, since the design and the production of (sub-)millimeter scale samples is much easier; thanks to the used wavelengths, which are essentially longer than in the optical range.5 Studies on transmission properties of terahertz radiation through periodic structures have been reported for different experimental approaches.6–10 In this paper we present results obtained for the transmission of broadband terahertz radiation through one-dimensional periodic structures of metallic cylinders whose separations within the structures are small compared to the wavelength. Still we observe a considerable transmission through the structure extending over a wide frequency range with dominant absorption peaks at the frequencies of SPPs11 excited by the terahertz pulses. The overall transmission is enhanced with respect to the ratio between the slit width and the period of the structures.12 The particular shape of the absorption peaks can be understood in terms of an interference between the resonant and the nonresonant contribution leading to typical Fano profiles.13,14 The resonant part is caused by the SPPs propagating along the surface, which are diffracted normal to the grating, whereas the nonresonant part is the enhanced direct transmission through the subwavelength slits between the cylinders (see Fig. 1).

For a metal mesh the spectral behavior in the vicinity of this resonance was described by Ulrich15 as a forced oscillation

\[
\tilde{t}(\nu) = t + \frac{r}{r} \frac{\alpha}{\alpha + i(\nu_{sp} - \nu)},
\]

where \(t\) and \(r\) are the amplitude transmission and reflection in the absence of the interaction of resonant and nonresonant parts, \(\alpha\) is the attenuation, and \(\nu_{sp}\) is the frequency of the surface plasmon, respectively. By changing the width of the slits, i.e., the spacing between the cylinders, the ratio between the resonant and nonresonant contributions can be tuned causing a variation of the shape of the Fano profiles.

The scattering of an incoming TE-wave from a periodic row of metallic cylinders with perpendicular incidence can be regarded as a two-dimensional (2D) problem.4 Starting from the homogeneous Maxwell equation and assuming the boundary condition for a perfect conductor

\[
\tilde{J}_S = \hat{n} \times \tilde{H}, \text{ and } 0 = \hat{n} \times \tilde{E}
\]

\((J_S\) are the currents on the surface, \(\tilde{H}\) is the magnetic field, and \(\tilde{E}\) is the electric field), one gets to the magnetic field equation (MFIE)16 for the incident magnetic field \(\tilde{H}^{inc}\) as follows:

\[
\hat{n} \times \tilde{H}^{inc} = \tilde{J}_S - (\hat{n} \times \nabla \times \hat{A})_{st},
\]

where

![FIG. 1. (Color online) Schematic illustrating the interference between direct transmission of terahertz radiation and scattered surface plasmon polaritons leading to Fano profiles.](image-url)
\[ A = J \ast G = \int \vec{J}(\vec{r}) \cdot G(\vec{r}) d\vec{r} \]

is the magnetic vector potential and
\[ G(\vec{r}) = \frac{1}{4i} H_0^{(2)}(k |\vec{r}|) \]

is the Green-function for the 2D-case.

Because not all boundary conditions have been taken into account, this equation lacks uniqueness. Therefore, it is assumed that the magnetic field inside the scatterer vanishes
\[ \hat{n} \times \vec{H}^{mc}|_{S^+} + i\hat{n} \times \vec{H}^{mc}|_{S^-} = \vec{J}_S - \{ \hat{n} \times \nabla \times \vec{A} \}_S^\pm, \]

where \( S^+ \) and \( S^- \) stand for an evaluation at a point infinitesimal outside or inside the scatterer, respectively.

To get the surface currents \( \vec{J} \) the MFIE is discretized by the method of moments on the surface of the scatterer. If the surface currents are known, the magnetic field \( H \), and the scattering cross section can be calculated easily.

For discretization the MFIE is written as \( \vec{\partial} \vec{J} = \vec{\beta} \) and the solution can be approximated by
\[ J = \sum_{n=1}^{N} J_n B_n, \]

where \( \{ B_n \} \) are basis functions and \( \{ J_n \} \) are unknown coefficients. They are obtained by forcing the residuum to be orthogonal to a set of testing functions \( \{ T_m \} \), which leads to
\[ l_{mn} = (T_m, B_n), \]

and the matrix coefficients are obtained because of the ease of evaluation and the continuity of the solution. Using \( \delta \)-functions as testing functions this simplifies to only one-dimensional integrals.

With the Floquet theorem \( J(x+\delta) = J(x)e^{-ik\delta} \), the simulation of an infinite periodic cylinder row with the periodicity \( \delta \) is simplified into a simulation of only one unit cell containing one single cylinder. Thus, the 2D Green-function is replaced with a periodic Green-function
\[ \vec{G}_p(x,y) = \frac{1}{4i} \sum_{q=\infty}^{\infty} H_0^{(2)}(k \sqrt{(x+|\delta|-q)^2+y^2}). \]

The sum of the Hankel functions \( H_0^{(2)} \) is slowly converging with \( O(n^{-1/2}) \). To accelerate the convergence a combination of Poisson and Kummers-transformation leads to a convergence rate of \( O(n^{-3/2}) \) and an exponential convergence rate of \( O(\exp(-|\vec{k}|y)) \) in the \( y \)-direction perpendicular to the periodicity. An adaptive algorithm calculates the optimal number of sum elements to be evaluated for every matrix element.

To compare the simulation with the experimental results the 2D bistatic scattering cross section (BSCS) is calculated and multiplied by the wavelength, which gives a quantity for the scattered spectral amplitude. The 2D cylinder (circle) was discretized by at least 20 points per wavelength and the BSCS was calculated for frequencies from 1 to 1100 GHz in steps of 1 GHz.

The experimental setup consists of a standard terahertz time-domain spectroscopy system with an InAs surface emitter and a photoconductive switch as the detector, described, e.g., in Refs. 17 and 18. Single-cycle broadband terahertz pulses centered around 600 GHz were used. Excitation of SPPs was performed in the collimated terahertz beam (diameter of 50 mm), as shown in Fig. 2. A typical periodic structure consists of a series of parallel metallic rods with diameters \( d \) between 0.5 and 2.5 mm. The rods were mounted in a machined plastic holder, so the distances were fixed by grooves that have been produced with a precision of \( \pm 2 \mu m \). The spacing between the cylinders can be tuned from direct contact up to 200 \( \mu m \) in steps of 5 \( \mu m \). The effective size of the structure was 60 x 80 mm\(^2\). The transmitted electric field was measured behind the structure with a spectral resolution of 4 GHz in a range from 45 to 1100 GHz. The spectrum was calculated via a fast Fourier transform.

A typical transmission spectrum for a row of cylinders with a diameter of 1.5 mm in “direct contact” is shown in Fig. 3 along with the data obtained from the numerical simulation. There is an excellent agreement of the position and line shape with respect to experimental parameters such as variation of the rod diameters and distances due to manufacturing accuracy.

The shape of the individual feature depends on the phase between the resonant diffracted and the directly transmitted contribution at the particular frequency. This can be seen in Fig. 3 where the first and second order resonances (200 and 400 GHz) exhibit Fano profiles while the third and fourth orders only show an absorption feature. This is due to the phase dependent interference with an increasing phase for higher frequencies. The fifth order resonance at 1 THz already shows a transition to a Fano profile with an inverted shape. Further higher order features with the completely in-
verted Fano profiles can be found in Fig. 2 in Ref. 14.

A considerable transmission starts at a frequency of approximately 120 GHz with an overall amplitude transmission of 10% in the observed frequency range up to 2 THz. If one assumes a maximum gap between the rods of 4 µm due to geometrical irregularities of the cylinders the ratio between gap width and cylinder diameter is approximately 0.003. The observed transmission corresponds to a geometrical enhancement factor for the intensity of more than 3.

This enhanced transmission can be explained in terms of a strong field enhancement in the gaps as described by Janunts et al.19 for touching cylinders. Here we concentrate on the shape of the absorption peaks in the spectra, which are caused by excitation of SPPs.14 The position of each peak is in perfect agreement with the calculated surface plasmon frequencies for perpendicular incidence. The corresponding wave vectors $\vec{k}_{\text{SPP}}$ are given by integer multiples $m$ of the inverse grating vector $\vec{G}$ as follows:

$$\vec{k}_{\text{SPP}} = m\vec{G} = m\frac{2\pi}{R}. \tag{1}$$

The shape of the Fano profiles can be modified by changing the gap width and thereby changing the resonant and nonresonant contributions to the transmission. This is illustrated in Fig. 4 where the experimentally determined relative transmission around the absorption peak corresponding to the second order SPP is shown for different gap widths. The larger the spacing between the cylinders gets, the less pronounced is the Fano profile. Once the gap width reaches the order of the wavelength, no Fano profiles can be observed anymore.

The frequency axis is normalized to the resonance frequency of the excited second order SPP. It is obvious from this figure that the shape of the resonance changes with the spacing. A comparison with our numerical simulation is displayed in Fig. 5 for a separation of 10 µm between the cylinders. The results obtained from the model of a forced oscillation16 are also included. This model, as well as our numerical simulation, shows a very good agreement with the experimental data. The performance of our numerical simulations extends over the whole frequency range whereas the forced oscillator model only describes the resonance structures. The residual deviation between experiment and simulation is caused by manufacturing accuracy of the structure and the higher spectral resolution of the simulation.

In summary, the transmission characteristics of pulsed broadband terahertz radiation through a one-dimensional periodic structure of metallic cylinders were investigated in detail. The excitation of SPPs leads to characteristic minima at resonance frequencies, which correspond to the frequencies of Wood’s anomalies identical to multiples of the inverse grating vector. The appearance of the observed minima was explained in terms of an interference between direct transmission of terahertz pulses and diffracted surface plasma polaritons giving rise to Fano profiles. Their shape was influenced by the ratio of resonant and nonresonant contributions. Numerical simulations using classical electrodynamics show excellent agreement with the experimental data.

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