# Lectures:

**17.1.2022** Recapitulation. First order and continuous phase transitions in the Van-der-Waals equation of state.

**19.1.2022** Critical exponents. Lattice gas model. Ising model.

# Book: Schwabl 5.4, 7.2

# Reading: „On water, steam, and string theory“

# Excercises: Please hand in until Mo 24.1.2022, 8:00 (15 points).

21) The phase transition line for sublimination from ice to steam is determined by the condition that the Gibbs Free Energy per particle $\mu = G(T,p)/N$ is equal for the gas and the solid. This can be estimated from known results as follows:

a) For the gas phase it is justified to assume the classical expression $\mu = k_B T \ln z = k_B T \ln N / Z_1$ from chapter 4.3, where $Z_1 = \frac{V}{\lambda_T^3} Z_{rot} \approx \frac{V}{\lambda_T^3} \frac{T}{\theta_{rot}}$ and $\theta_{rot} \approx 22K$. Find the corresponding analytical expression for $\mu = G(T,p)/N$ as a function of $T$ and $p$.

b) For the solid phase, assume phonons can be described by the Debye model from chapter 3.10. Show that $F_{phonons} = -k_B T \int_0^{\omega_D} d\omega\, g(\omega) \ln Z(\omega) \approx N k_B T (3\ln(\theta_D / T) - 1)$ in the high temperature limit $\theta_D / T << 1$. Compare $G_{phonons} = F_{phonons} + pV$ with $G_{gas}$ from part a) using $\theta_D = 192K$, $\rho_{ice} \approx 0.9 g/cm^3$ and the molecular mass of water 18g/mol. Argue that $G_{phonons} / G_{gas}$ is small for pressures $p$<1000Pa.

c) The phase transition can be determined by setting the chemical potential from the ideal gas equal to the Gibbs Free Energy per particle in the solid phase $G(T,p) = E_{bind} + G_{phonons}$, where $E_{bind} = -51 kJ/mol$ and $G_{phonons}$ is negligible. Determine a corresponding expression for the pressure of the phase transition as a function of $T$. Do the values at $T$=223K and $T$=273K roughly agree with the plot in chapter 5.2, page 5?