Information for the exam on the 10.2.2022 at 12:00:

There will be 1-2 questions from each chapter as listed below. Each chapter counts 20 points. Answers can be given in German or English and should cover the same detail as given on the lecture slides.

Grades: <40 **fail**, 40-55: **4**, 56-70: **3**, 70-84: **2**, 85-100: **1**

Chapter 1

1.) What is an exact differential? What is an integrating factor? Give one example each using thermodynamic quantities.

2.) What do the zeroth and first law of thermodynamics say?

3.) What is the second law of thermodynamics?

4.) What is a Legendre transformation? Give an example from thermodynamics.

5.) Give the thermodynamic definitions for the free energy F, the enthalpy H, the free enthalpy G. What are the respective differentials? Give an example of how pressure can be calculated as a derivative of a suitable thermodynamic potential.

6.) What is a generalized force? Give an example!

7.) Derive one Maxwell relation.

8.) Describe the Carnot cycle and sketch the corresponding p-V and S-T diagrams.

9.) Calculate the efficiency for reversible cycle processes. (Hint: you may use the Carnot cycle).

Chapter 2

1.) What is the definition of the entropy of a general probability distribution?

2.) What is the assumption for using the microcanonical ensemble. What is the probability distribution in the microcanonical ensemble?

3.) When are two systems in energetic equilibrium in the microcanonical ensemble? What does this mean for entropy?

4.) Define temperature in the microcanonical ensemble.

5.) How are generalized forces calculated in the microcanonical ensemble?

6.) Calculate the entropy of an ideal classical gas in 3D using the micro-canonical ensemble (except for the prefactor). Derive the ideal gas law. Show that the average energy per particle is $E=3k_BT/2$.

Chapter 3

1.) What are the assumptions for using the canonical ensemble?

2.) What is the canonical partition function? Derive the expectation values of the energy as an expression of the partition function.

3.) How is the free energy defined? How can energy, entropy and pressure be calculated from the canonical partition function (or the free energy)?

4.) What is a single-particle partition function and when is it useful?

5.) Apply the methods of the canonical ensemble to the ideal classical gas. Calculate the expectation values for pressure, energy and entropy.

6.) Show in general that energy fluctuations in the canonical ensemble can be determined by specific heat and temperature.

7.) How is the single-particle density of states $g(\varepsilon)$ defined?

8.) Derive the single-particle density of states for the case of a free particle with mass m in three dimensions.

9.) Calculate the partition function, the expectation energy value and the specific heat for a harmonic quantum oscillator.

10.) What is the Einstein model?

Chapter 4

1.) Define the grand canonical partition function, the grand canonical potential, the fugacity z and the chemical potential.

2.) What are the assumptions for the grand canonical ensemble and what is the corresponding probability for a state with N particles and energy E?

3.) Apply the concept of the grand canonical ensemble to an ideal classical gas with single particle partition function $Z_1 = V/\lambda_T^3$. What is the fugacity z? Find an expression for E and p as a function of Z_1, T, V and N.

4.) Give general expressions for the logarithm of the grand canonical partition function of non-interacting bosons and fermions. (Hint: you can use a sum over single-particle states).

5.) Derive the Bose-Einstein and the Fermi-Dirac distribution from the corresponding non-interacting grand canonical partition functions.

6.) How are the particle number, the energy, the pressure and the entropy expressed as a sum over single-particle states for non-interacting bosons and fermions, respectively?

7.) How is the Fermi energy ε_F defined? Calculate the pressure and energy for an ideal three-dimensional Fermi gas at T=0 as a function of N, V and ε_F. (Hint: use the corresponding single particle density of states).

Chapter 5

1.) Explain how a phase transition between an ordered and a disordered phase can be understood qualitatively by minimizing the free energy.

2.) What characterizes a first-order phase transition? What is meant by an order parameter? What is the Ehrenfest Classification?

3.) What is latent heat?

4.) What is the Clausius Clapeyron equation? Derive it.

5.) What is the Van der Waals equation of state? What is the meaning of the parameters a and b? Sketch $p(V)$ qualitatively for temperatures above and below the critical temperature.

6.) What is a critical point? Discuss using an example.

7.) What is Curie's law? Derive it for a spin-1/2 magnetic moment.

8.) Explain cooling by adiabatic demagnetization.

9.) Consider domain walls in the Ising model. Argue that they appear at any temperature in a 1D system in the thermodynamic limit.

10.) Explain the third law of thermodynamics? (Nernst theorem).

11.) Describe the basic idea and steps of a mean field theory for a *general* interacting model.