Chapter 5.9: The Ising model in one dimension with magnetic field

$$H_{\text{Ising}} = -J\sum_{J} S_j S_{j+1} - B\sum_{j} S_j$$

Definition of the transfer matrix

$$Z = \sum_{\{S_j\}} e^{-\beta H} = \sum_{\{S_j\}} e^{\beta\sum_{j}(JS_jS_{j+1}+BS_j)} = \sum_{\{S_j\}}\prod_{j} e^{\beta JS_jS_{j+1}+\beta\frac{B}{2}(S_j+S_{j+1})}$$

$$Z = \mathrm{tr}T^N$$ where $$T = \begin{pmatrix} e^{\beta(J+B)} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta(J-B)} \end{pmatrix}$$

Eigenvalues of the Transfer matrix

$$U^T T U = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$ where

$$\lambda_{1/2} = e^{\beta J} \cosh \beta B \pm \sqrt{e^{-2\beta J} + e^{2\beta J} \sinh^2 \beta B}$$

Exact partition function

$$Z = \mathrm{tr} T^N$$

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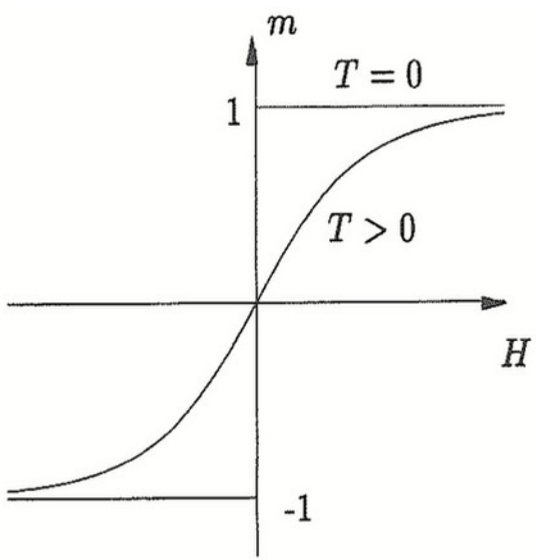
Magnetization and susceptibility

$$M = -\frac{\partial F}{\partial B} = N\frac{\sinh \beta B}{\sqrt{\sinh^2 \beta B + e^{-4\beta J}}}$$



$$C_H = -T\partial^2 f/\partial T^2$$