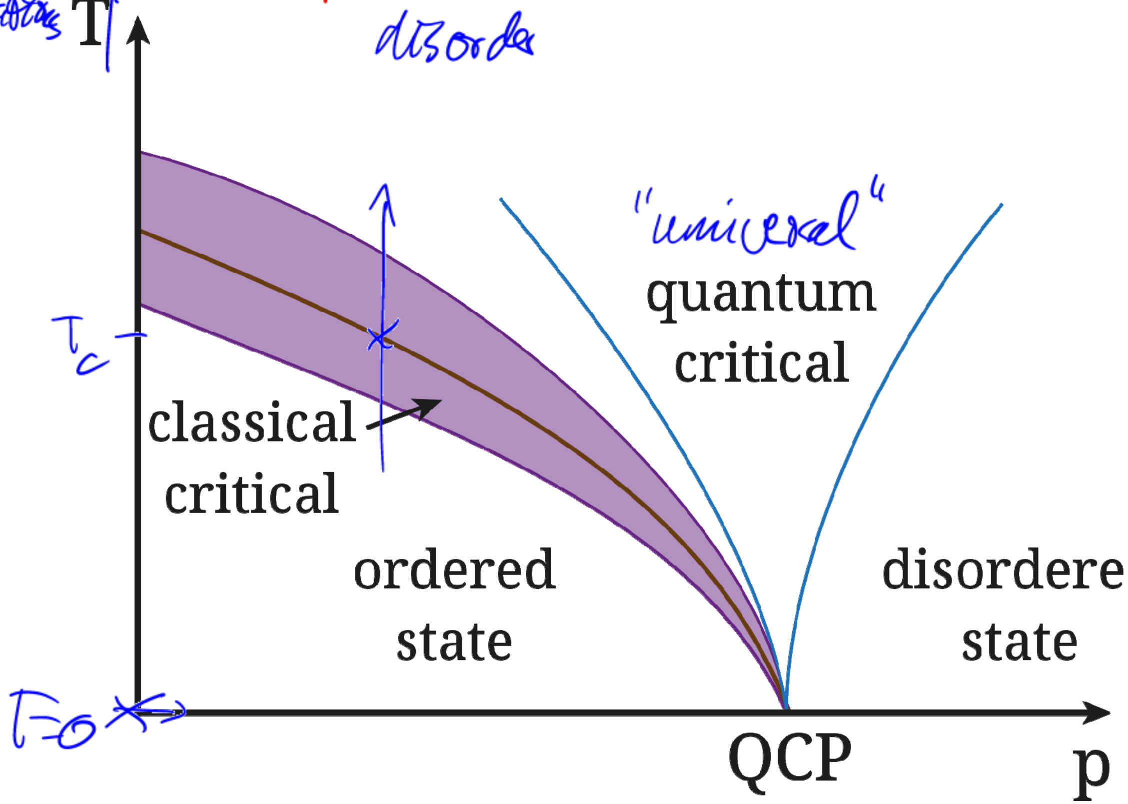
Chapter 5.14: Quantum phase transitions

Changing a parameter may increase quantum fluctuations → transition to a disordered state

from non-commuting operators

Thermal fluctuations

disorder



$Z = \mathrm{tr}\, e^{-\beta H}$

Propagator

$U(t) \sim \langle x | e^{-iHt} | x' \rangle$

in imaginary

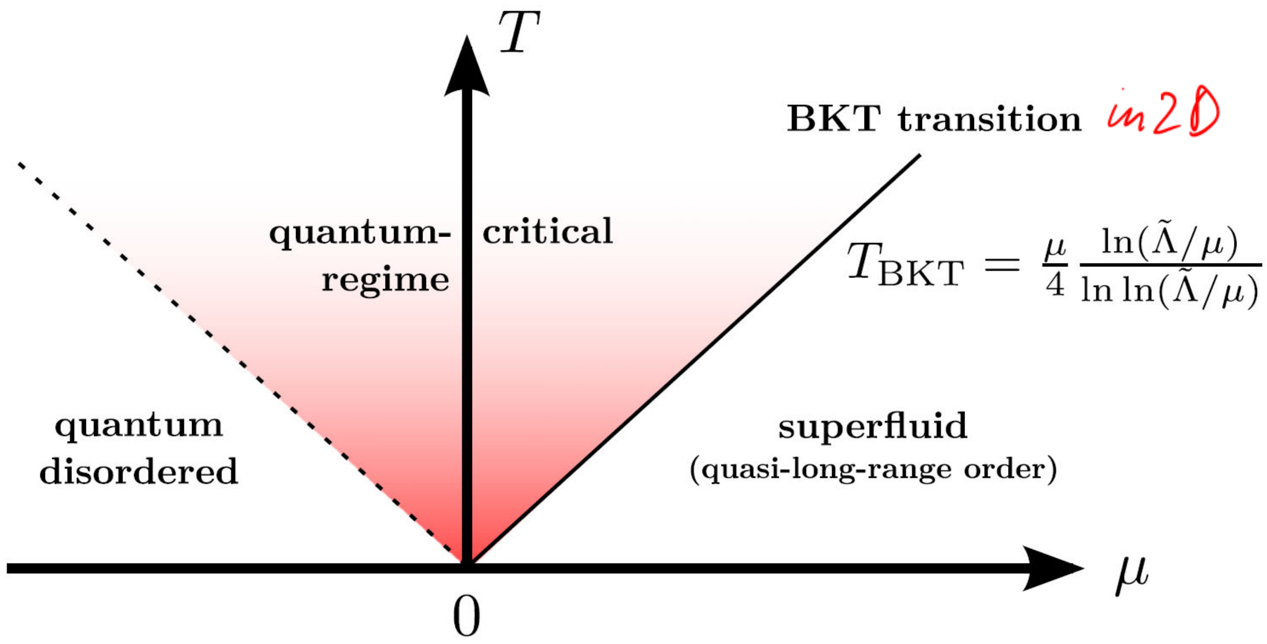
density, pressure

field

doping

* interactions!

Most basic example: weakly interacting bosons



XY universality

ϕ^4- model

superfluid transition

coupled spin chains

Relation to XY-model: Holstein Primakoff Transformation

$$\hat{S}_j^z = S - \hat{n}_j \,,$$

$$\hat{S}_j^+ = \sqrt{2S - \hat{n}_j}\,\hat{b}_j \,, \approx b_j \sqrt{2S} \quad S^+ = S^x + iS^y$$

$$\hat{S}_j^- = \hat{b}_j^\dagger \sqrt{2S - \hat{n}_j} \,, \approx \sqrt{2S}\, b_j^\dagger \quad S^- = S^x - iS^y$$

exact mapping of the SU(2) relation

$[S^x, S^y] = i S S^z$ "SU(2)" commutation relation

XY model

$$H = \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y) = \frac{1}{2} \sum_{\langle ij \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$$

$$= S \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + H_{int}$$

order

$\langle S_x \rangle \neq 0$

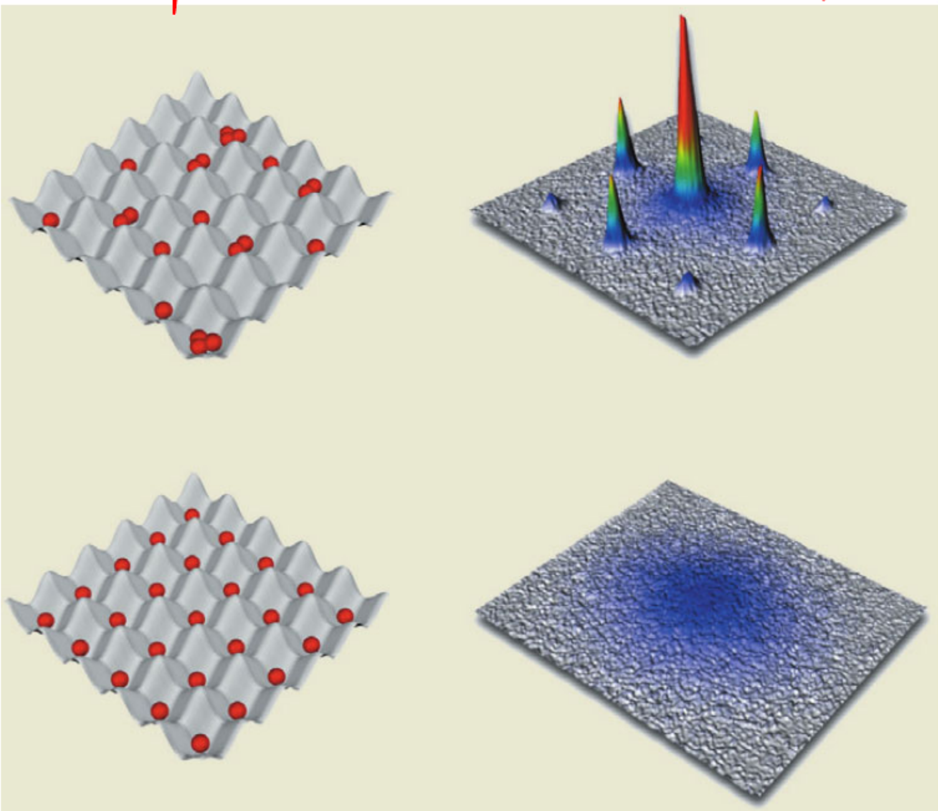
$\Rightarrow \langle b \rangle \neq 0$ off diagonal long range order

The Bose Hubbard model

e.g. optical lattice

$$\hat{H}_{\mathrm{BH}} = -J \sum_{\langle i,j\rangle \in \Lambda^2} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_{i\in\Lambda} \hat{n}_i(\hat{n}_i - 1) - \mu \sum_{i\in\Lambda} \hat{n}_i.$$

in real space   interference experiment



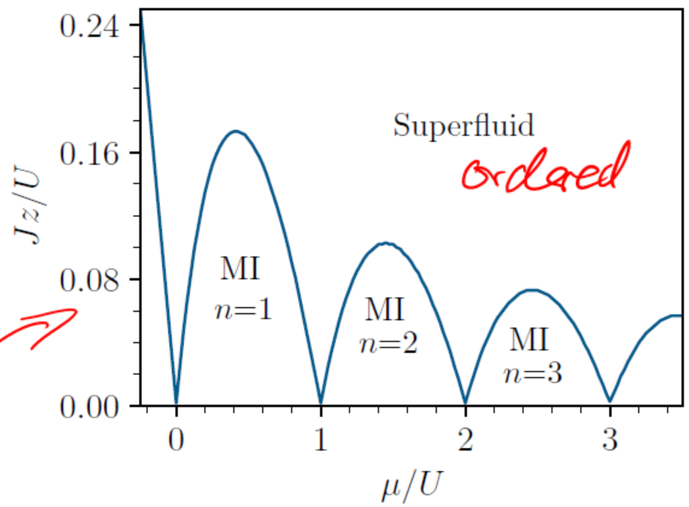
$J \gg U$ BEC
→ coherent superfluid state
$\langle a \rangle \neq 0$
"ordered state"

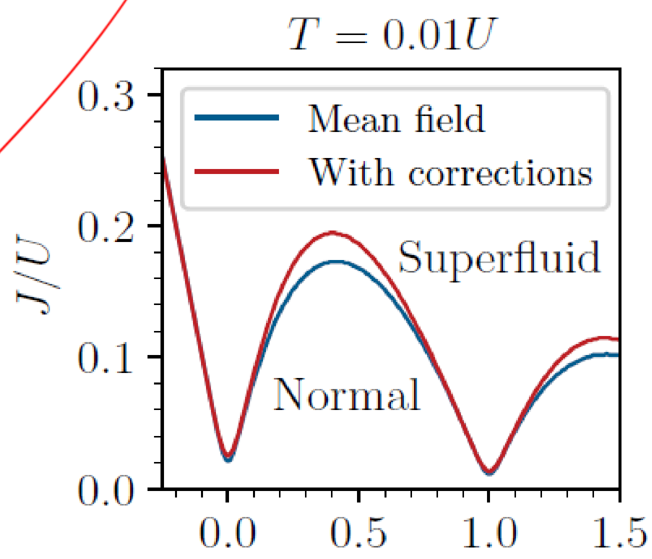
$J \ll U$ "Mott" state
insulator
incoherent → "disordered state"

5.14-4 Quantum phase transitions

The Bose Hubbard model at finite temperatures and experiment and theory

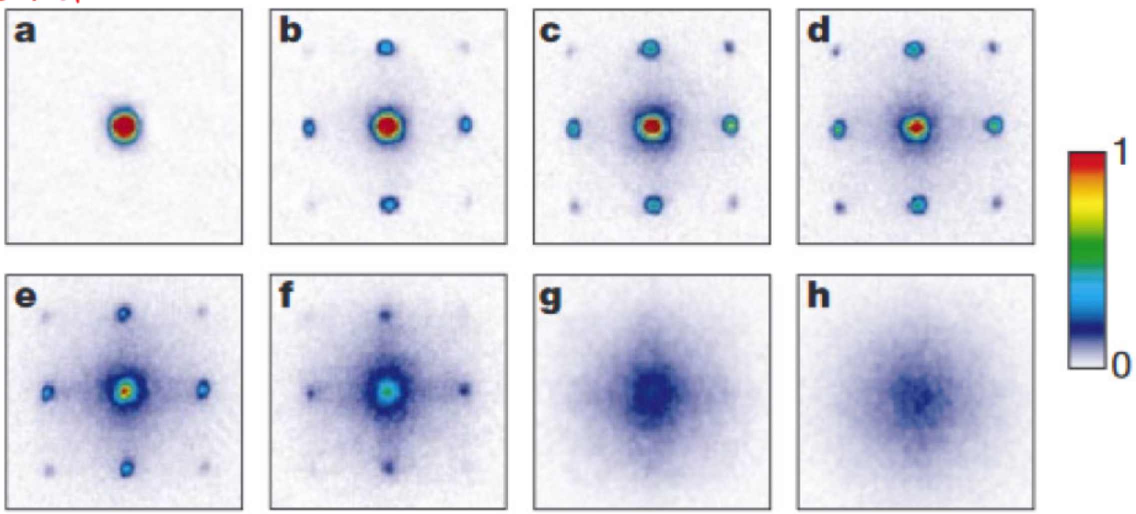
Mean field: order parameter $\psi = \langle a \rangle$





$$\hat{H}_{\mathrm{MF}} = \sum_{i\in\Lambda}\left\{-zJ\left(\psi^*\hat{a}_i + \psi\hat{a}_i^\dagger - |\psi|^2\right) + \frac{U}{2}\hat{n}_i\left(\hat{n}_i - 1\right) - \mu\hat{n}_i\right\}$$

ordered



disordered

Greiner et al. (2003)