Chapter 5.11: Mean field theory

Problem: coupled degrees of freedom (for example: $H_{\text{Ising}} = -J\sum_{\langle i,j\rangle} S_i S_j - B\sum_j S_j$)

General **mean field approach** for two coupled degrees of freedom $\widehat{\Omega}$ and $\hat{\Lambda}$: $H = \widehat{\Omega}\hat{\Lambda}$

1.) neglect fluctuations of order $(\widehat{\Omega} - \langle\widehat{\Omega}\rangle)(\hat{\Lambda} - \langle\hat{\Lambda}\rangle)$

$$H = \widehat{\Omega}\hat{\Lambda} \approx \widehat{\Omega}\hat{\Lambda} - \left(\widehat{\Omega} - \langle\widehat{\Omega}\rangle\right)\left(\hat{\Lambda} - \langle\hat{\Lambda}\rangle\right) = \widehat{\Omega}\langle\hat{\Lambda}\rangle + \langle\widehat{\Omega}\rangle\hat{\Lambda} - \langle\widehat{\Omega}\rangle\langle\hat{\Lambda}\rangle$$

2.) self-consistently determine mean fields $\langle\widehat{\Omega}\rangle$ and $\langle\hat{\Lambda}\rangle$

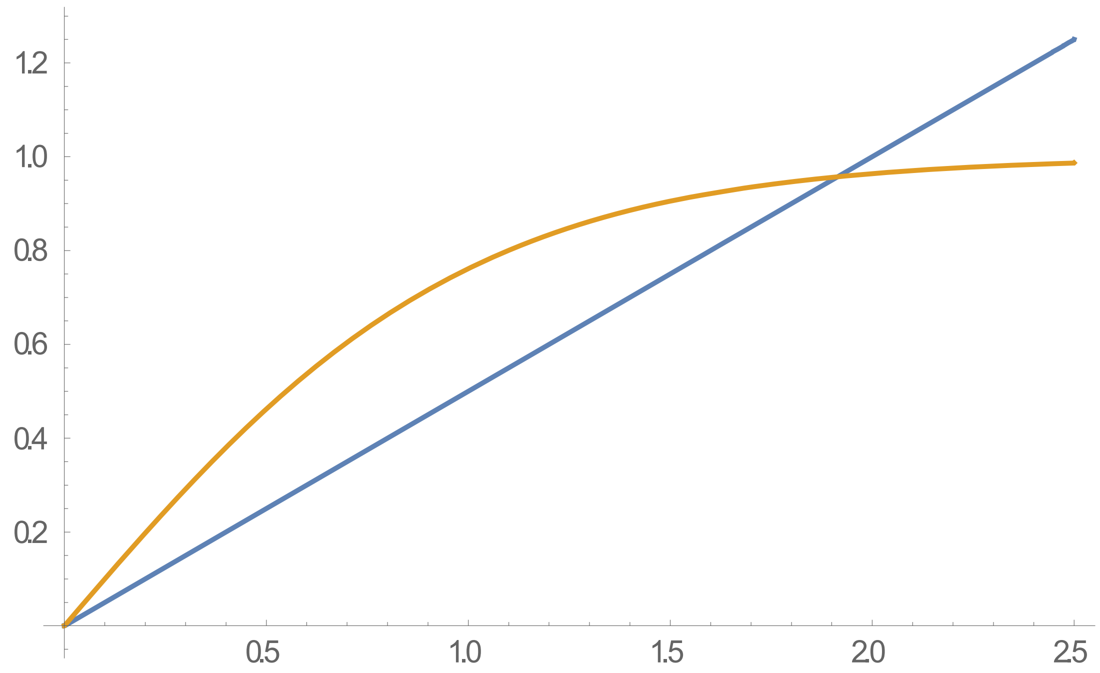
Application to Ferromagnet:

$$H_{\text{Ising}} = -J\sum_{\langle i,j\rangle} S_i S_j - B\sum_j S_j \quad \approx \quad -B_{eff}\sum_j S_j$$

where $B_{eff} = B + zJ\langle S_j \rangle$

self-consistency:

$$\langle S_j \rangle = m(B_{eff}) = \tanh\,\beta B_{eff}$$



Mean field susceptibility:

$$B_{eff} = B + zJm$$

$$\chi = \frac{\partial m(B_{eff})}{\partial B} = \chi_0 \frac{\partial B_{eff}}{\partial B} = \chi_0 \left(1 + zJ \frac{\partial m(B_{eff})}{\partial B} \right)$$

General MFT result:

$$\chi \approx \frac{\chi_0}{1 - zJ\chi_0}$$

Curie Weiss law (exact for high temperatures):

$$\chi = \frac{j(j+\frac{1}{2})}{3k_B T} \left(1 + \frac{T^c_{MFT}}{T} \right)$$

Improvements: Chain mean field and Bethe lattice