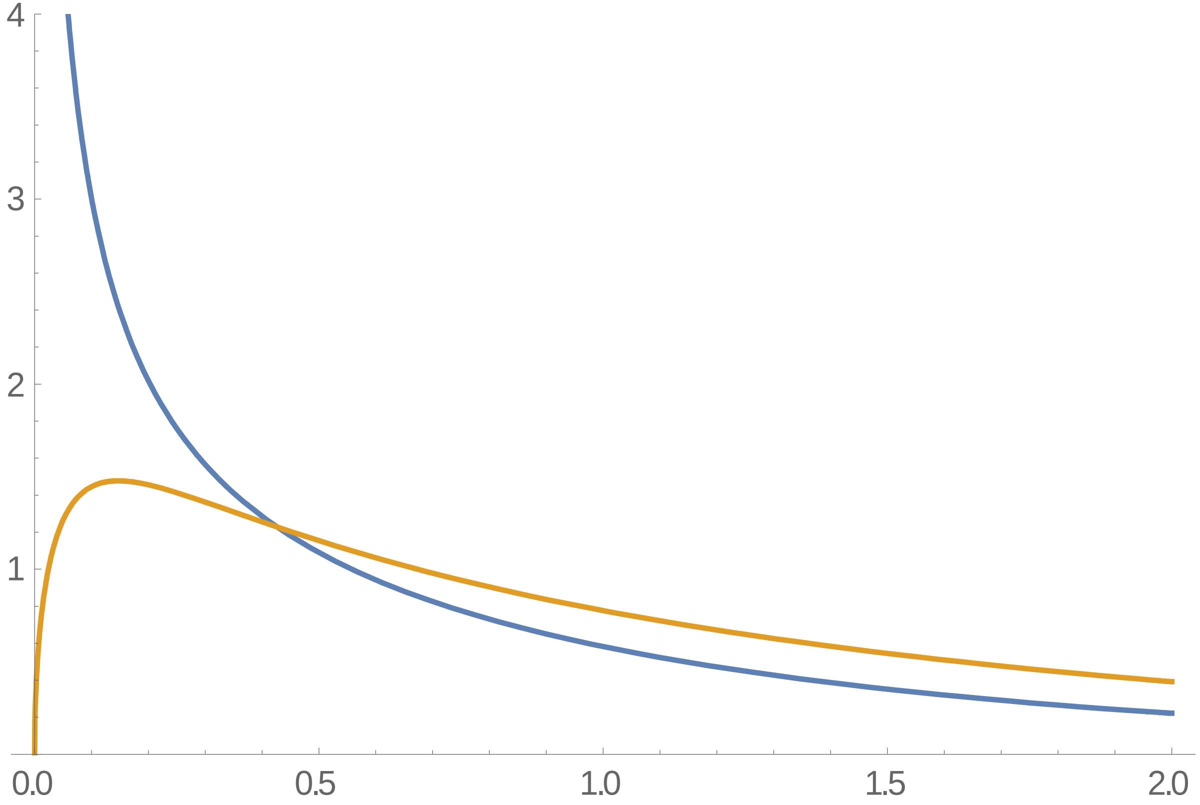
Chapter 4.7: The Bose Einstein Condensation

$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{z^{-1} e^{\beta\varepsilon} - 1} = \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}(z)$$

$$\varepsilon = (n_x^2 + n_y^2 + n_z^2) \frac{\hbar^2 \pi^2}{2mL^2}$$



$$\lambda_T = \hbar\sqrt{\frac{2\pi}{mk_BT}}$$

Corrected number calculation: ground state separately

$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \langle n_0 \rangle + \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{z^{-1}e^{\beta\varepsilon} - 1} = \frac{1}{z^{-1} - 1} + \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}(z)$$

$x_0 = \dfrac{\langle n_0 \rangle}{N} = \dfrac{1}{N}\dfrac{1}{z^{-1}-1}$ becomes finite as $z \rightarrow 1$ and $N \rightarrow \infty, \quad V \rightarrow \infty$

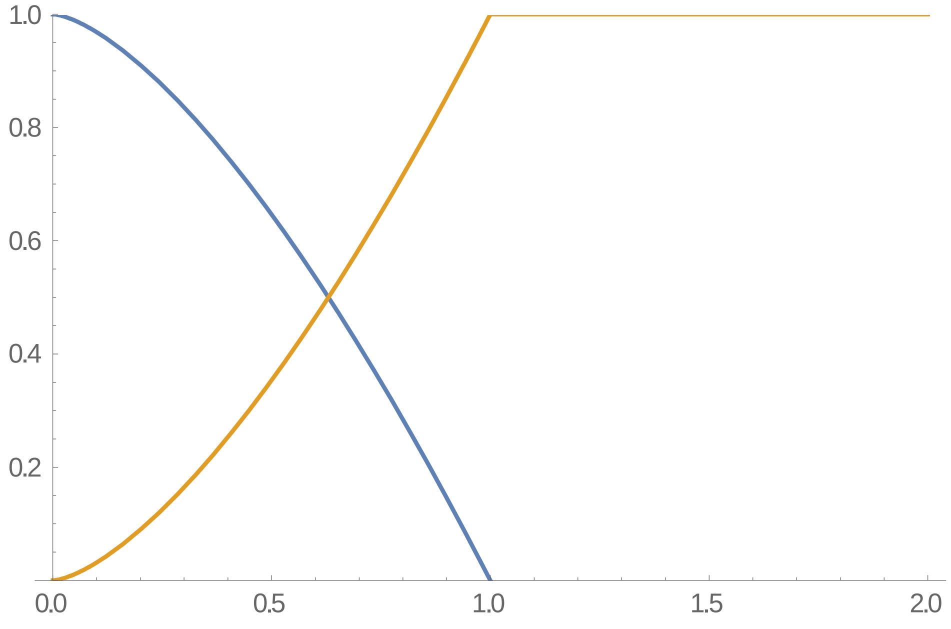
Temperature at which ground state occupation x_0 becomes significant is

$$N = \frac{V}{\lambda_{T_c}^3} \mathrm{Li}_{3/2}(1) = \frac{V}{\lambda_{T_c}^3} \zeta\left(\tfrac{3}{2}\right)$$

For $T > T_c$: $$N = \frac{V}{\lambda_T^3}\mathrm{Li}_{3/2}(z)$$

$$\frac{V}{N} = \frac{\lambda_{T_c}^3}{\zeta(3/2)}$$

For $T < T_c$: $$N = \frac{1}{z^{-1}-1} + \frac{V}{\lambda_T^3}\mathrm{Li}_{3/2}(1)$$



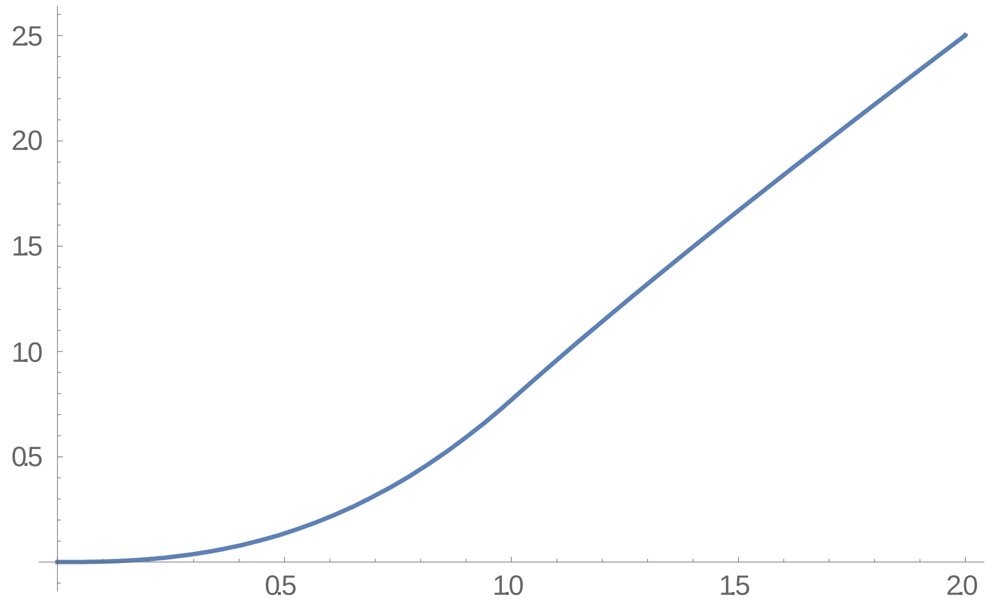
$$\frac{V}{N} = \frac{\lambda_{T_c}^3}{\zeta(3/2)}$$

Consequences for energy and pressure

$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad = \frac{\varepsilon_0}{z^{-1} - 1} \quad + \int_0^\infty d\varepsilon \, \frac{\varepsilon \, g(\varepsilon)}{z^{-1} e^{\beta\varepsilon} - 1} = \frac{3}{2} \frac{k_B T V}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

$$p = -\left(\frac{\partial \Phi}{\partial V} \right)_{\alpha, T} \quad \rightarrow \quad \frac{k_B T}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

4.7-5 The Bose Einstein Condensation



Consequences for specific heat

$$c_V = -\left(\frac{\partial E}{\partial T}\right)_V$$