Chapter 4.6: Low density expansion for Bosons

Energy, Particle Number, Pressure

$$\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}$$

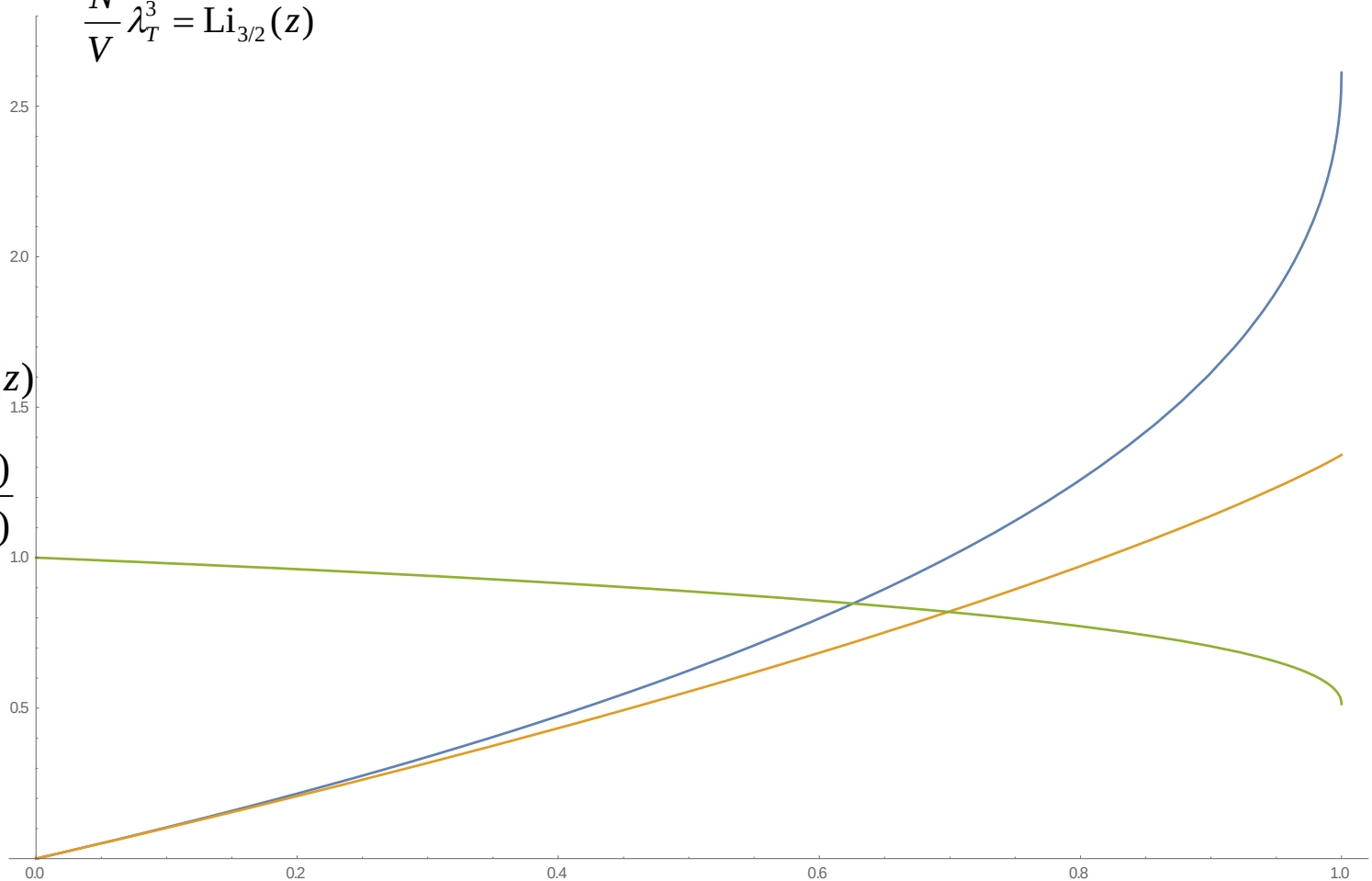
$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}(z)$$

$$\frac{N}{V} \lambda_T^3 = \mathrm{Li}_{3/2}(z)$$

$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad \rightarrow \quad \frac{3}{2} \frac{k_B T V}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

$$= \frac{3}{2} k_B T N \frac{\mathrm{Li}_{5/2}(z)}{\mathrm{Li}_{3/2}(z)}$$

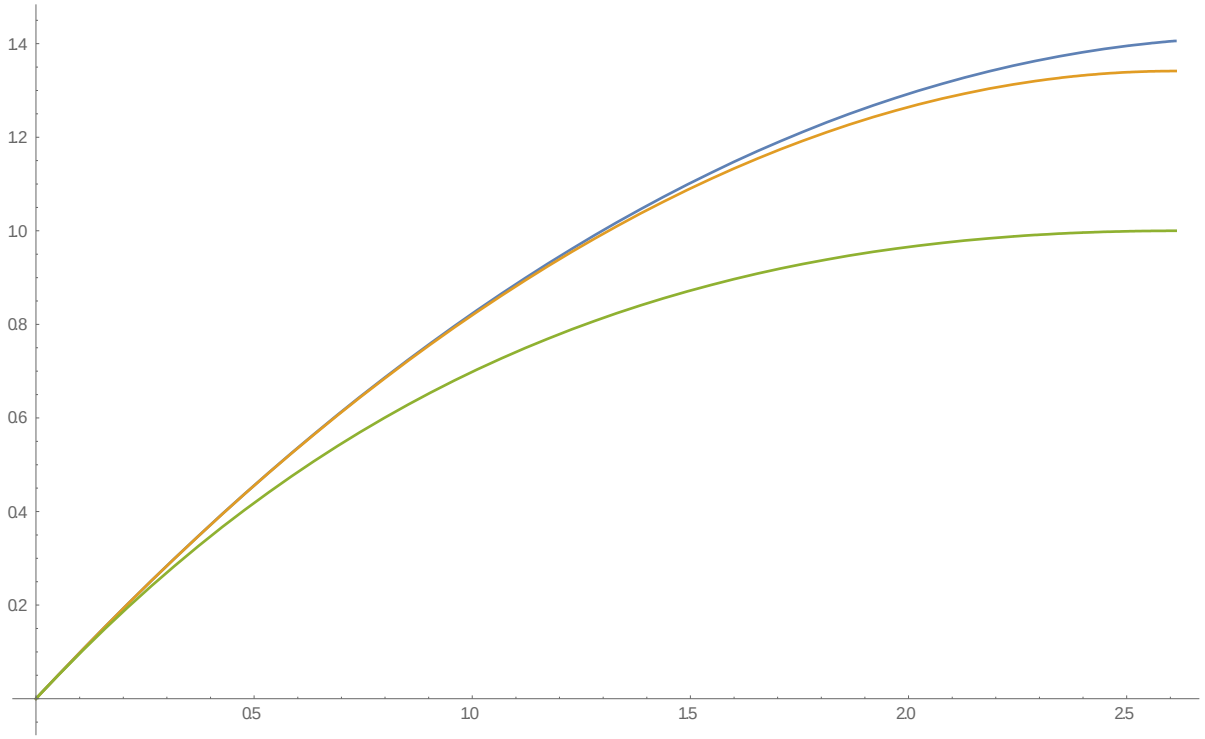
$$p = -\frac{\Phi}{V} \quad \rightarrow \quad \frac{k_B T}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$



$$\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}$$

Virial expansion: Inverse series

$$\frac{N}{V}\lambda_T^3 = \mathrm{Li}_{3/2}(z)$$



$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad \rightarrow \quad \frac{3}{2}\frac{k_B T V}{\lambda_T^3}\mathrm{Li}_{5/2}(z)$$

4.6-3 Low density expansion for Bosons

$$\lambda_T = \hbar\sqrt{\frac{2\pi}{mk_BT}}$$

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InverseSeries[Series[PolyLog[3/2, x], {x, 0, 3}]]
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$$x - \frac{x^2}{2\sqrt{2}} + \frac{1}{36}\left(9 - 4\sqrt{3}\right)x^3 + O[x]^4$$

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PolyLog[5/2, InverseSeries[Series[PolyLog[3/2, x], {x, 0, 3}]]]
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$$x - \frac{x^2}{4\sqrt{2}} + \left(\frac{1}{8} - \frac{2}{9\sqrt{3}}\right)x^3 + O[x]^4$$

Specific heat

$$c_V = -\left(\frac{\partial E}{\partial T}\right)_V$$

Critical Temperature at which $z \to 1$: $N = \frac{V}{\lambda_{T_c}^3}\mathrm{Li}_{3/2}(1) = \frac{V}{\lambda_{T_c}^3}\zeta(\tfrac{3}{2})$