$$Z_r = \frac{1}{1 - z e^{-\beta \varepsilon_r}}$$

Chapter 4.6: The Bose Gas in three dimensions

$$\ln Z_B = -\sum_r \ln(1 - z e^{-\beta \varepsilon_r}) \quad \to \quad -\int_0^\infty d\varepsilon \, \ln(1 - z e^{-\beta \varepsilon}) g(\varepsilon)$$

Single particle density of states in 3D in Chpt. 3.7

$$g(\varepsilon) = \sum_r \delta(\varepsilon_r - \varepsilon) = \frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3} \sqrt{\varepsilon}$$

$$x = \beta \varepsilon$$

$$\ln Z_B \to -\frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3} \frac{1}{\beta^{3/2}} \int_0^\infty \underbrace{\sqrt{x}}_{u'} \, \underbrace{\ln(1 - z e^{-x})}_{v} \, dx$$

part. integration

$$= \frac{V}{\lambda_T^3} \frac{2}{\sqrt{\pi}} \int_0^\infty \underbrace{\frac{2}{3} x^{3/2}}_{u} \underbrace{\frac{z e^{-x}}{1 - z e^{-x}}}_{v'} + \cancel{\text{boundary terms}} \to 0$$

$$\lambda_T = \hbar \sqrt{\frac{2\pi}{m k_B T}}$$

$$= \frac{4}{3\sqrt{\pi}} \frac{V}{\lambda_T^3} \int_0^\infty \frac{x^{3/2}}{e^x z^{-1} - 1} dx = \frac{V}{\lambda_T^3} Li_{\frac{5}{2}}(z)$$

4.6-2 The Bose Gas in three dimensions

Polylogarithm functions

$$\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{z^{-1}e^x - 1}\, dx$$

$$\Gamma(\nu) = (\nu-1)!$$

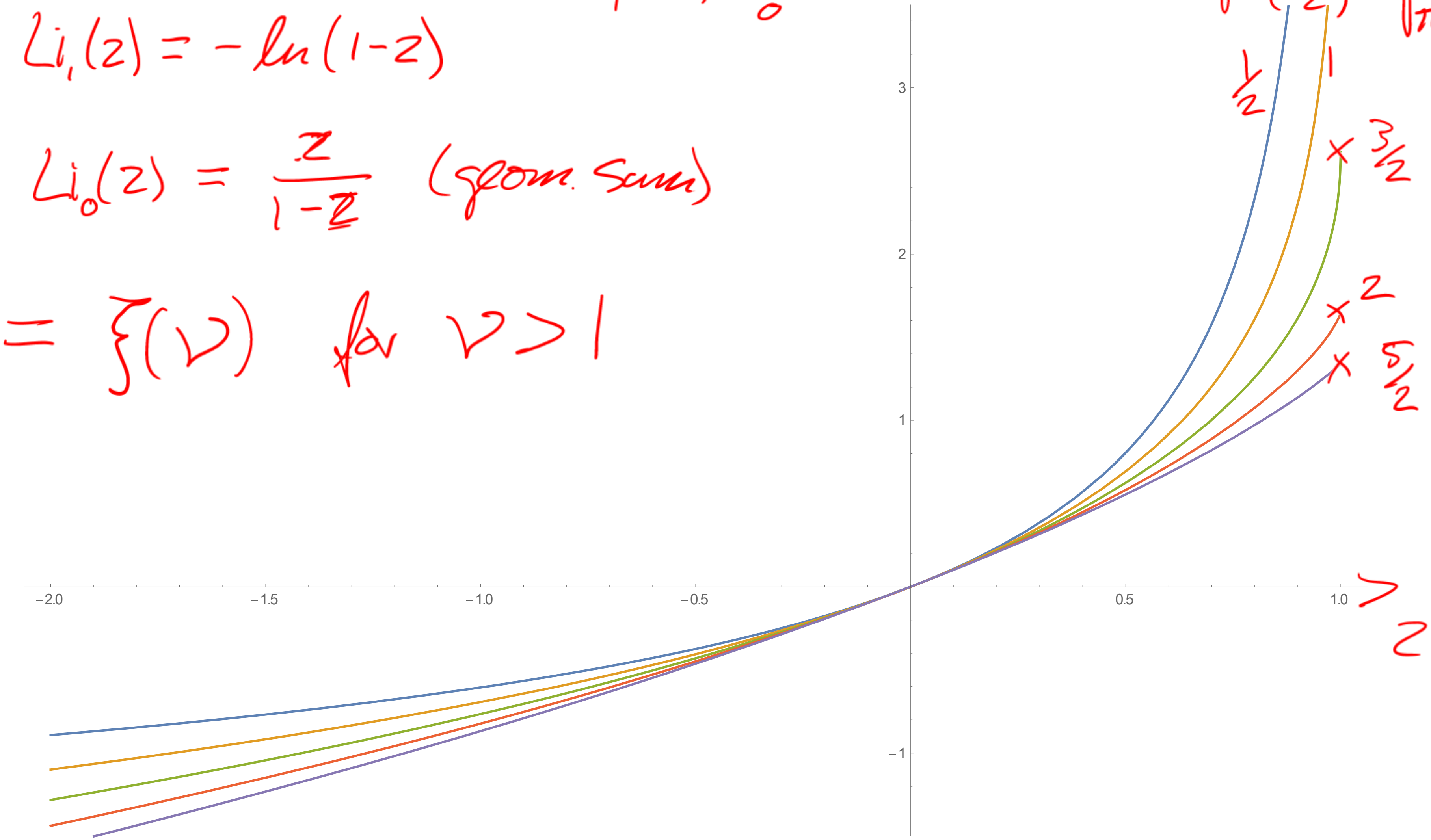
$$\Gamma\left(\tfrac{5}{2}\right) = \tfrac{3}{4}\sqrt{\pi}$$

$$\Gamma\left(\tfrac{3}{2}\right) = \frac{2}{\sqrt{\pi}}$$

for $\nu = 1$ $\quad \mathrm{Li}_1(z) = -\ln(1-z)$

for $\nu = 0$ $\quad \mathrm{Li}_0(z) = \dfrac{z}{1-z}$ (geom. sum)

$$\mathrm{Li}_\nu(1) = \zeta(\nu) \quad \text{for } \nu > 1$$



$x = \beta \varepsilon$

Energy, Particle Number, Pressure

$$N = \sum_r \langle n_r \rangle \;\to\; \int_0^\infty d\varepsilon \frac{g(\varepsilon)}{z^{-1}e^{\beta\varepsilon} - 1} = \frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3} \frac{1}{\beta^{3/2}} \int_0^\infty dx \frac{\sqrt{x}}{z^{-1}e^{x} - 1} = \frac{V}{\lambda_T^3} Li_{3/2}(z)$$

$$z = Li_{3/2}^{-1}\left(\frac{\lambda_T^3 N}{V}\right)$$

$$E = \sum_r \varepsilon_r \langle n_r \rangle \;\to\; \int_0^\infty d\varepsilon \frac{\varepsilon\, g(\varepsilon)}{z^{-1}e^{\beta\varepsilon} - 1} = \frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3} \frac{1}{\beta^{5/2}} \int_0^\infty dx \frac{x^{3/2}}{z^{-1}e^{x} - 1} = \frac{3}{2} k_B T \frac{V}{\lambda_T^3} Li_{5/2}(z)$$

$$p = -\frac{\Phi}{V} = \frac{k_B T \ln \mathcal{Z}}{V} = \frac{k_B T}{\lambda_T^3} Li_{5/2}(z) = \frac{2}{3} \frac{E}{V}$$

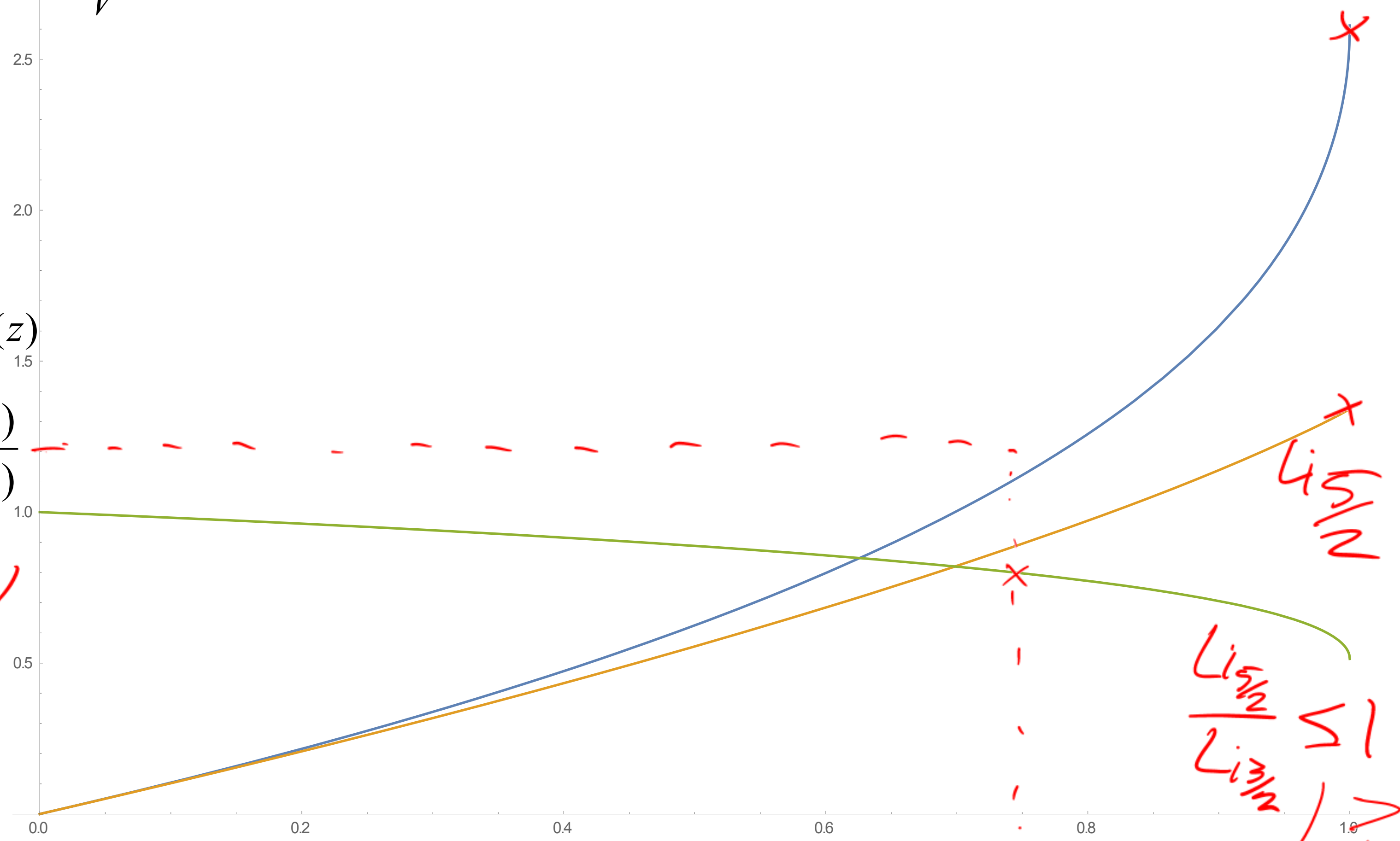
$$\underline{E = \frac{3}{2} pV}$$

also for ideal gas

Energy, Particle Number, Pressure

$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}(z)$$

$$\frac{N}{V}\lambda_T^3 = \mathrm{Li}_{3/2}(z)$$



$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad \rightarrow \quad \frac{3}{2}\frac{k_B T V}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

$$= \frac{3}{2} k_B T N \frac{\mathrm{Li}_{5/2}(z)}{\mathrm{Li}_{3/2}(z)}$$

$$\leq \frac{3}{2} k_B T N$$

$$p = -\frac{\Phi}{V} \quad \rightarrow \quad \frac{k_B T}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

- expansion in z or $\frac{N}{V}\lambda_T^3$
- what happens $\frac{N}{V}\lambda_T^3 > \zeta(\frac{3}{2})$???

energy, pressure are reduced, depends on V