Chapter 4.5: The Bose Gas in three dimensions

$$\ln \mathcal{Z}_{\mathrm{B}} = -\sum_r \ln(1 - z e^{-\beta\varepsilon_r}) \quad \rightarrow \quad -\int_0^\infty d\varepsilon \, \ln(1 - z e^{-\beta\varepsilon}) g(\varepsilon)$$

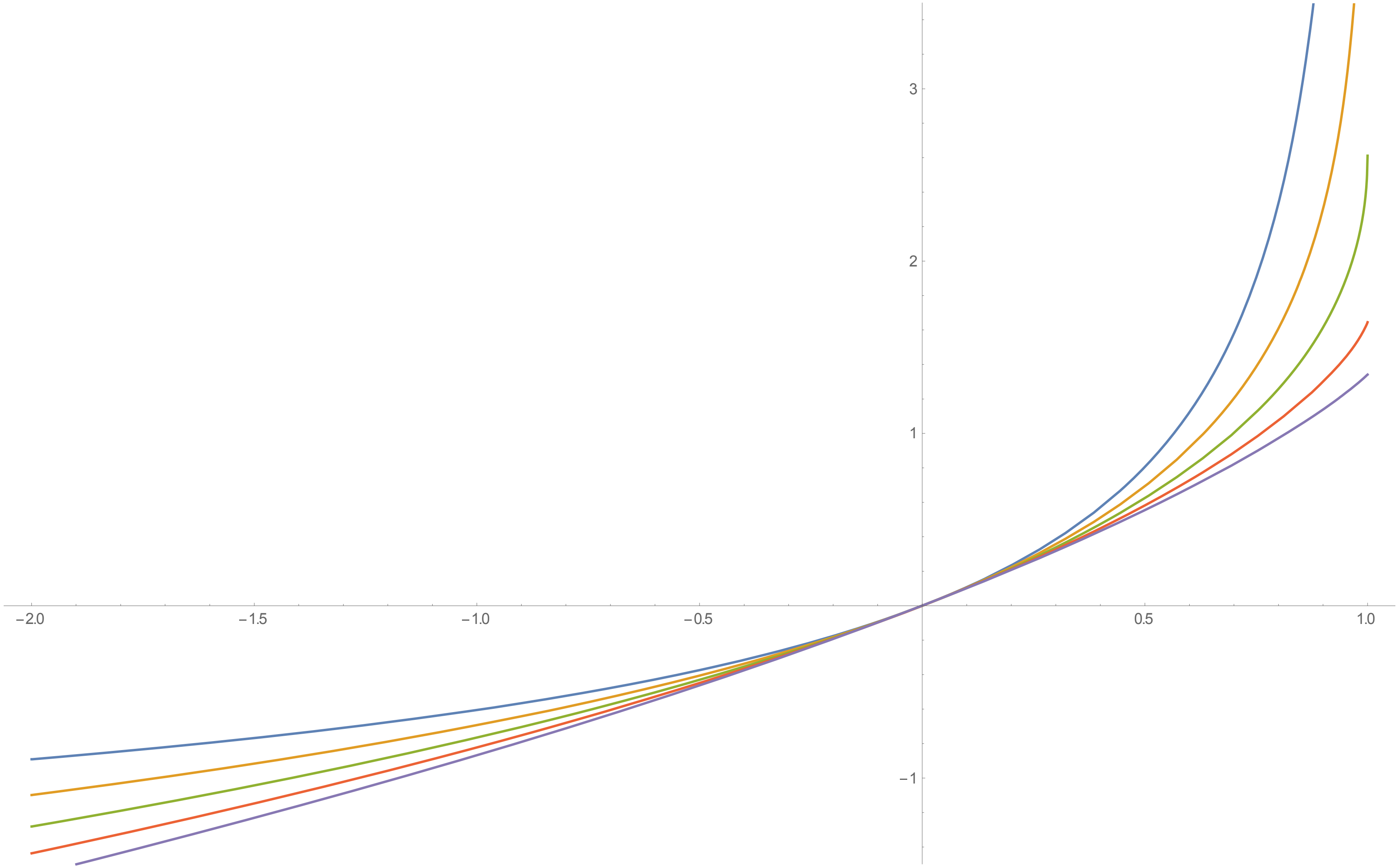
Single particle density of states in 3D

$$g(\varepsilon) = \sum_r \delta(\varepsilon_r - \varepsilon) = \frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3} \sqrt{\varepsilon}$$

$$\lambda_T = \hbar \sqrt{\frac{2\pi}{m k_B T}}$$

Polylogarithm functions

$$\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}$$



Energy, Particle Number, Pressure

$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{z^{-1} e^{\beta\varepsilon} - 1}$$

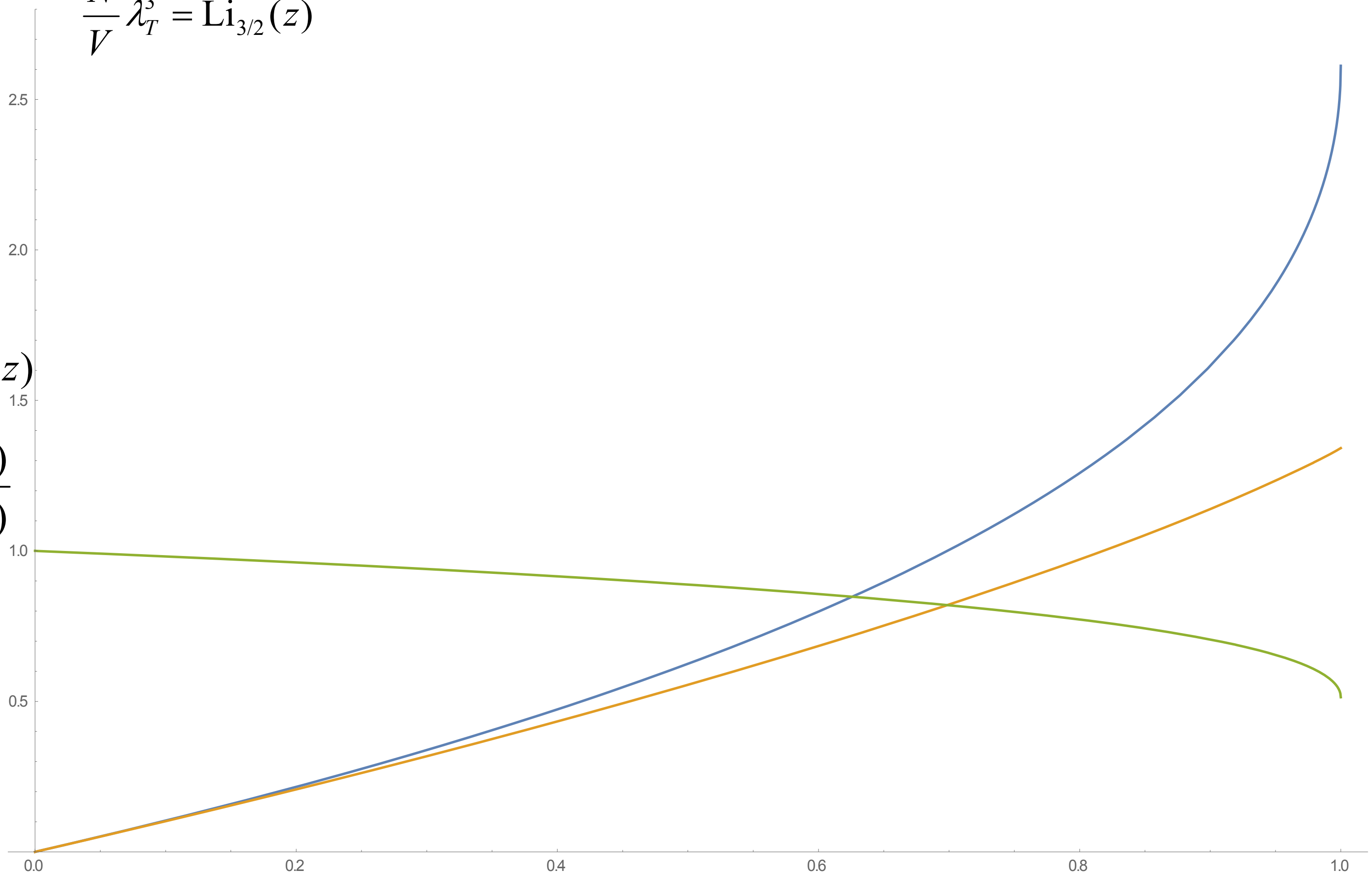
$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad \rightarrow \quad \int_0^\infty d\varepsilon \, \frac{\varepsilon \, g(\varepsilon)}{z^{-1} e^{\beta\varepsilon} - 1}$$

$$p = -\frac{\Phi}{V}$$

Energy, Particle Number, Pressure

$$N = \sum_r \langle n_r \rangle \quad \rightarrow \quad \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}(z)$$

$$\frac{N}{V} \lambda_T^3 = \mathrm{Li}_{3/2}(z)$$



$$E = \sum_r \varepsilon_r \langle n_r \rangle \quad \rightarrow \quad \frac{3}{2} \frac{k_B T V}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$

$$= \frac{3}{2} k_B T N \frac{\mathrm{Li}_{5/2}(z)}{\mathrm{Li}_{3/2}(z)}$$

$$p = -\frac{\Phi}{V} \quad \rightarrow \quad \frac{k_B T}{\lambda_T^3} \mathrm{Li}_{5/2}(z)$$