Chapter 4.3 Indistinguishable Quantum particles

Indistinguishable particles are not independent

Bosons: superposition of states is symmetric under exchange

Fermion: superposition of states is antisymmetric under exchange

Example: two particles in three states

4.3-3 Indistinguishable quantum particles

We expect significant quantum effects if the
number of accessible states is not large compared to the number of parcticles

Classical

Quantum

For ideal gas of independent quantum particles:

Number of accessible states is given by single particles states up to temperature $k_B T$

$$\int_0^{k_B T} d\varepsilon\, g(\varepsilon)$$

Single particle density of states

$$g(\varepsilon) = \sum_r \delta(\varepsilon_r - \varepsilon) = \frac{m^{3/2} V}{\sqrt{2}\pi^2 \hbar^3}\sqrt{\varepsilon}$$

Thermal wave length $\lambda_T = \hbar \sqrt{\dfrac{2\pi}{m k_B T}}$

4.3-5 Indistinguishable quantum particles

Canonical probability $P_r = \frac{e^{-\beta\varepsilon_r}}{Z_1}$ to find a single particle in the microstate r is related to probability that this state is occupied.

Summary: Each basis state is fully symmetrized/anti-symmetrized superposition.

Impossible to specify which particle is in which state. Instead specify
occupation numbers n_r = number of particles in state $|r\rangle$

Basis states:

Bosons

$$|n_{r_1}, n_{r_2}, n_{r_3}, n_{r_4}, ...\rangle_B$$

Fermions

$$|n_{r_1}, n_{r_2}, n_{r_3}, n_{r_4}, ...\rangle_F$$