Chapter 4.2: The Ideal Gas in the Grand Canonical Ensemble

Can we recover the known ideal gas law in the grand canonical ensemble?

Previous results: $Z_N = \frac{Z_1^{\,N}}{N!}$ and $Z_1 \approx \frac{V}{\lambda_T^3}$ $\qquad \lambda_T = \hbar\sqrt{\frac{2\pi}{mk_BT}}$

$$\mathbf{Z} = \sum_{N=0}^{\infty} e^{-\alpha N} \sum_{\{r\}} e^{-\beta E(\{r\},N)} = \sum_{N=0}^{\infty} z^N Z_N$$

1.) Find relation between N and $\mu = -k_B T \alpha$

$$N = -\left(\frac{\partial \Phi}{\partial \mu}\right)_T = -\left(\frac{\partial \ln \mathbf{Z}}{\partial \alpha}\right)_\beta$$

2.) Determine rest

$$E = -\left(\frac{\partial \ln \mathbf{Z}}{\partial \beta}\right)_{\alpha,V}$$

$$p = -\left(\frac{\partial \Phi}{\partial V}\right)_{\alpha,T}$$