Chapter 3.4: Density of states and velocity distribution

Basis: Boltzmann distribution $P_r = \frac{e^{-\beta\varepsilon_r}}{Z}$

Is the probabilty to find a given microstate. But what is probability $P(\varepsilon)$ to find energy ε?

$$E = \frac{\int d\Gamma\, e^{-\beta\varepsilon}\varepsilon}{Z}$$

For a single particle: $\langle \varepsilon \rangle = \dfrac{\sum_r e^{-\beta\varepsilon_r} \varepsilon_r}{Z_1}$

Def. 3.6: The single particle density of states is $g(\varepsilon) = \sum_r \delta(\varepsilon_r - \varepsilon)$

The single particle density of states $g(\varepsilon)$, determines the probability $P(\varepsilon)d\varepsilon = g(\varepsilon)e^{-\beta\varepsilon} d\varepsilon / Z_1$ for energies in the interval $[\varepsilon, \varepsilon + d\varepsilon[$ for each independent particle.

Example: Single particle density of states for a free classical particle

$$\left\langle \varepsilon \right\rangle = \frac{\int d\varepsilon\, g(\varepsilon) e^{-\beta\varepsilon} \varepsilon}{Z_1} = \frac{\int d^3\vec{r} d^3\vec{p}\; e^{-\beta\varepsilon} \varepsilon}{Z_1}$$

Distribution for general expectation values:

$$\langle \Lambda \rangle = \frac{\sum_r e^{-\beta \varepsilon_r} \langle r | \Lambda | r \rangle}{Z_1}$$

Classical example: velocity in x-direction

$$\langle v_x \rangle = \int dv_x \; P(v_x) v_x = \frac{\int d^3\vec{r} d^3\vec{p} \; e^{-\beta \varepsilon} v_x}{Z_1}$$

Maxwell's velocity distribution

$$\langle v \rangle = \int dv\, P(v) v = \frac{\int d^3\vec{r} d^3\vec{p}\; e^{-\beta\varepsilon} v}{Z_1}$$

Def. 3.7: Maxwell's velocity distribution

$$P(v)dv = 4\pi \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-\beta m v^2/2} dv$$

is the probability for a particle-velocity in the interval $[v, v+dv[$

Mean, most probable and RMS values

$$P(v)dv = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\beta m v^2/2} dv$$

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