Chapter 3.3: Classical degrees of freedom

classical microstate $\{\vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2, \vec{r}_3, \vec{p}_3, ..., \vec{r}_N, \vec{p}_N\}$, phase space integration: $\prod_j d^3\vec{r}_j d^3\vec{p}_j = d\Gamma^{6N}$

More general: $\{q_1, q_2, q_3, q_4, ...\}$ with energy $\varepsilon(q_1, q_2, q_3, q_4, ...)$ (not necessarily independent)

Integration: $d\Gamma = \prod_j dq_j$

Consider : $\left\langle q_j \frac{\partial \varepsilon}{\partial q_k} \right\rangle = \frac{1}{Z} \int d\Gamma \, q_j \frac{\partial \varepsilon}{\partial q_k} e^{-\beta\varepsilon}$

<u>Def. 3.4</u>: The <u>virial theorem</u> for statistical mechanics states, that for continuous classical variables

$$\left\langle q_j \frac{\partial \varepsilon}{\partial q_k} \right\rangle = \delta_{jk} k_B T$$

Special case: Bilinear energy $\varepsilon = \sum_{jk} A_{jk} q_j q_k$

Def. 3.5: The equipartition theorem for statistical mechanics states, that for an classical energy $\varepsilon_j = \sum_j \varepsilon_j$ with $\varepsilon_j = \sum_k A_{jk} q_j q_k$

$$\left\langle \varepsilon_j \right\rangle = \frac{1}{2} k_B T$$