Chapter 3.10: Blackbody radiation

Idea Planck (1900):

The radiation dependence on frequency ν can be explained if integral is replaced by sum

$$\langle \varepsilon(\nu) \rangle = \frac{\int_0^\infty d\varepsilon \, e^{-\beta\varepsilon} \varepsilon}{\int_0^\infty d\varepsilon \, e^{-\beta\varepsilon}} \rightarrow \frac{\sum_{n=0}^{\infty} e^{-\beta\varepsilon_n} \varepsilon_n}{\sum_{n=0}^{\infty} e^{-\beta\varepsilon_n}}$$

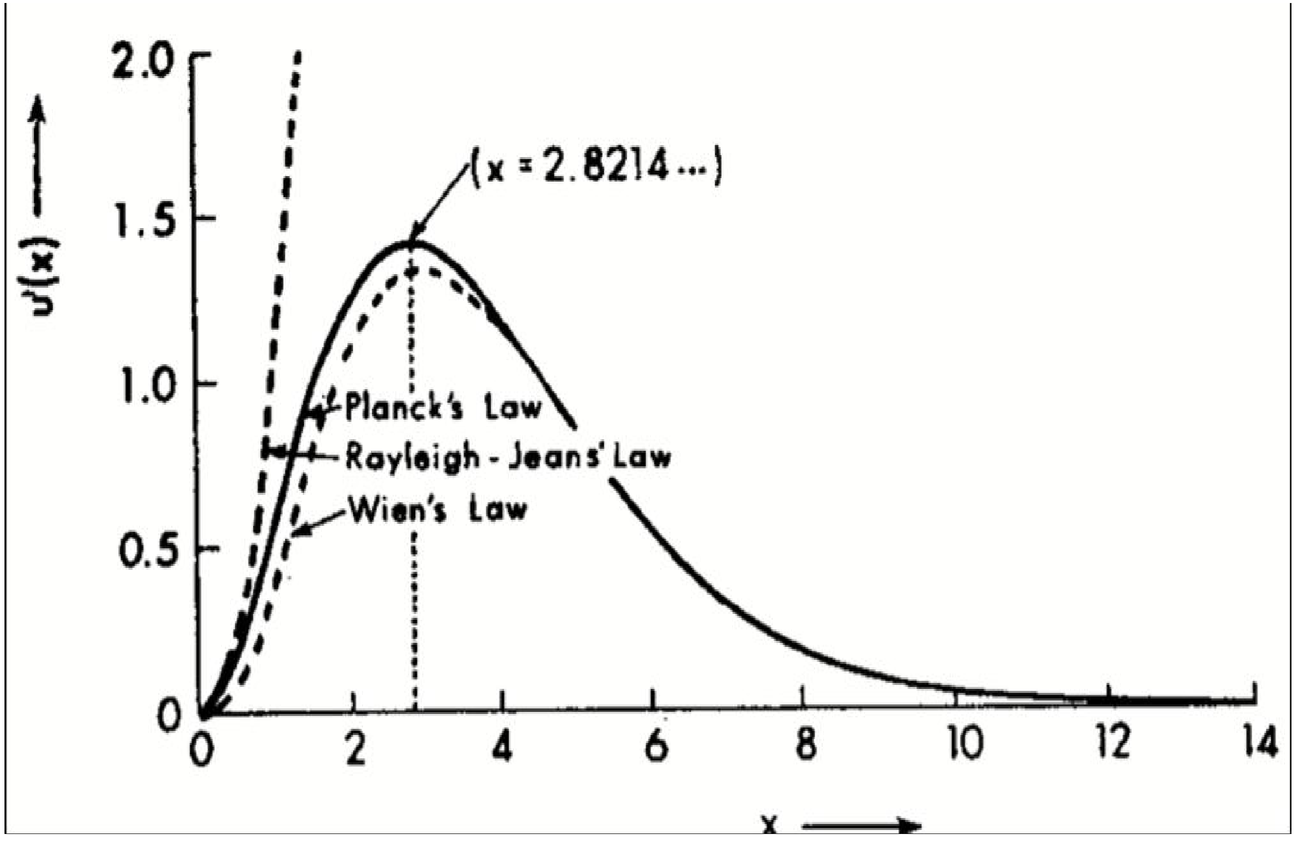
where $\varepsilon_n = hn\nu$

Energy per oscillation mode: $\langle \varepsilon(\omega) \rangle = \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1}$

Energy density is $\langle \varepsilon(\omega) \rangle g(\omega) d\omega$ where density of (frequency) states:

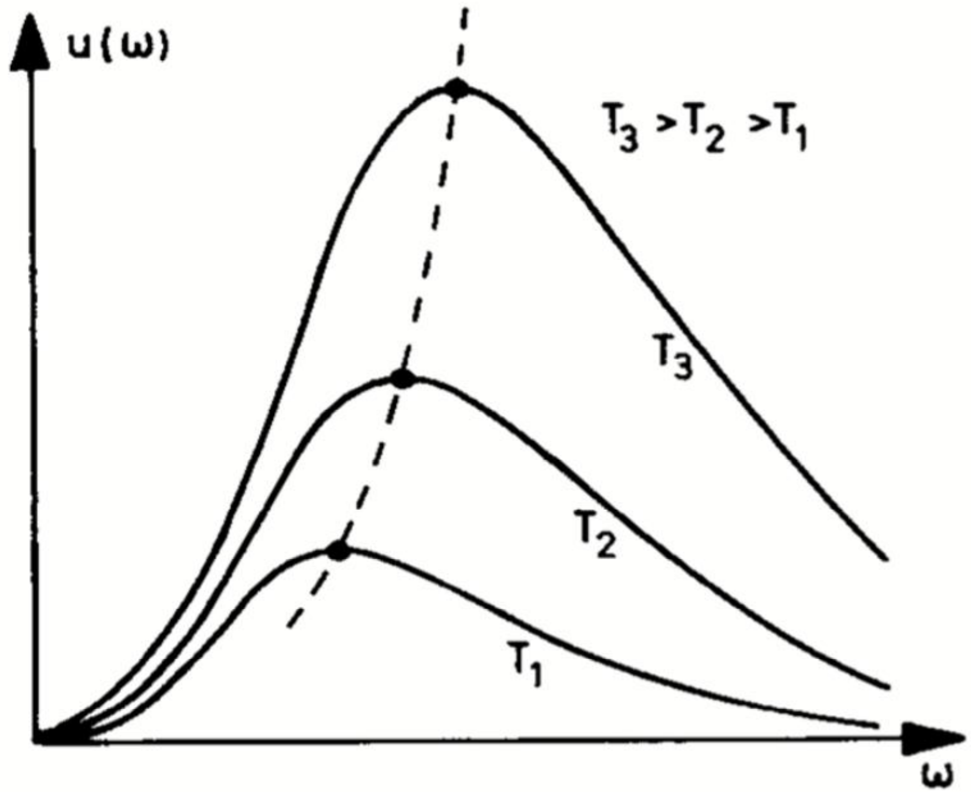
$$g(\omega) = \sum_{\vec{k},\alpha} \delta(\omega_{\vec{k},\alpha} - \omega)$$

$$\omega_{\vec{k},\alpha} = c\left|\vec{k}\right|$$



Total Energy: T^4 behavior

$$E = \int_0^\infty \langle \varepsilon(\omega) \rangle g(\omega) d\omega = \frac{V}{\pi^2 c^3} \int_0^\infty \frac{\hbar\omega^3}{e^{\beta\hbar\omega} - 1} d\omega$$



Partition function, entropy, and radiation pressure:

Partition function $Z_\omega = \sum_{n=0}^{\infty} e^{-\beta \varepsilon_n}$

$$\ln Z = \sum_{\alpha} \sum_{\vec{k}} \ln Z_{\vec{k},\alpha} \approx \int_0^\infty d\omega \, g(\omega) \ln Z_\omega$$

Entropie $S = k_B \ln Z + \frac{E}{T}$

Pressure $p = -\left(\frac{\partial F}{\partial V}\right)_T = k_B T \left(\frac{\partial \ln Z}{\partial V}\right)_T$

Einstein (1905), Bose (1925):
Energy quanta are real and can be described by indistinguishable massless particles

$\langle n(\omega) \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}$ is the number of energy quanta → number of particles