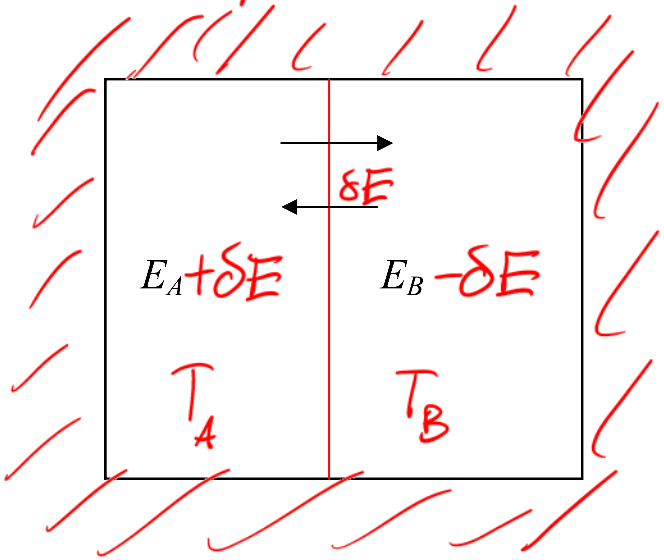
Chapter 2.6: Microcanonical Irreversibility

Equilibrium is "most probable" state

Consider the change of entropy under energy exchange

How likely is heat exchange, What is probability to find E_A, E_B



$$S = k_B \ln \Omega(E_A, E_B) = k_B \underline{\ln \Omega_A(E_A)\,\Omega_B(E_B)}$$

$$\delta S = k_B\, \delta(\ln \Omega_A + \ln \Omega_B)$$

$$= k_B \frac{\partial \ln \Omega_A}{\partial E_A} \delta E - \frac{\partial \ln \Omega_B}{\partial E_B} \delta E + O(\delta E^2)$$

$$= k_B (\beta_A - \beta_B)\, \delta E \quad + O(\delta E^2)$$

$$k_B \beta = \frac{1}{T}$$

Entropy increase

$$\delta S = \left(\frac{1}{T_A} - \frac{1}{T_B}\right)\delta E + \mathcal{O}(\delta E^2)$$

Number of states increases if $T_A < T_B$ and $\delta E > 0$

$$S = k_B \ln \Omega$$

$$\Omega = e^{S/k_B}$$

$$\Omega(E_A + \delta E, E_B - \delta E) = e^{\frac{S+\delta S}{k_B}} = \underbrace{\Omega(E_A, E_B)}_{\Omega_{before}}\, e^{\delta S/k_B}$$

Probability of change

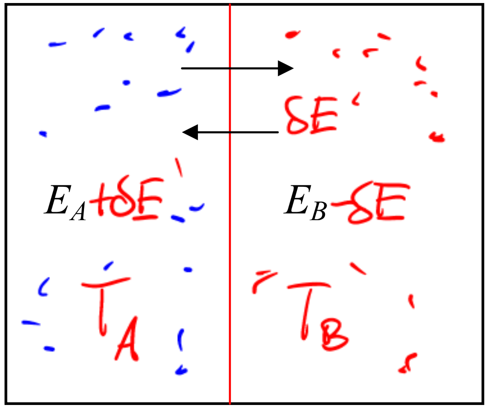
$$\frac{\Omega_{after}}{\Omega_{before}} = e^{\frac{\delta S}{k_B}} = e^{\frac{1}{k_B}\left(\frac{1}{T_A} - \frac{1}{T_B}\right)\delta E} = e^{(\beta_A - \beta_B)\delta E}$$

exponentially more likely to transfer heat from $T_A < T_B$

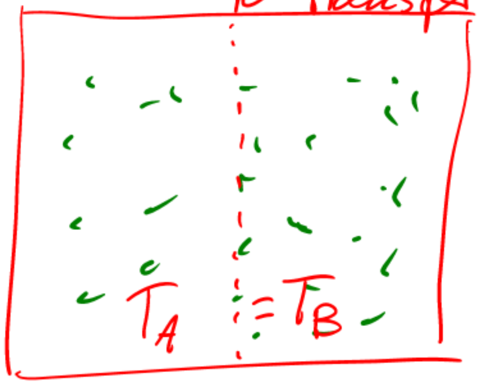
$\rightarrow$ 2nd law of thermodyn.

Summary: All microstates with the same total energy are equally probable, but overwhelmingly many are observed in equilibrium.

exponentially likely / unlikely to transfer heat



time



Arrow of time is "more likely"

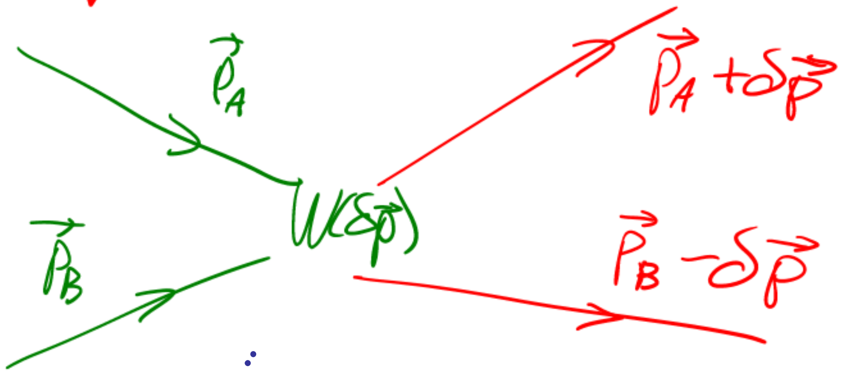
Boltzmann H-Theorem (1872)

assume time-reversible laws with scattering probability

with uncorrelated scattering events

derive that $S(t)$ increases in time

not a "general" proof



Critisism:

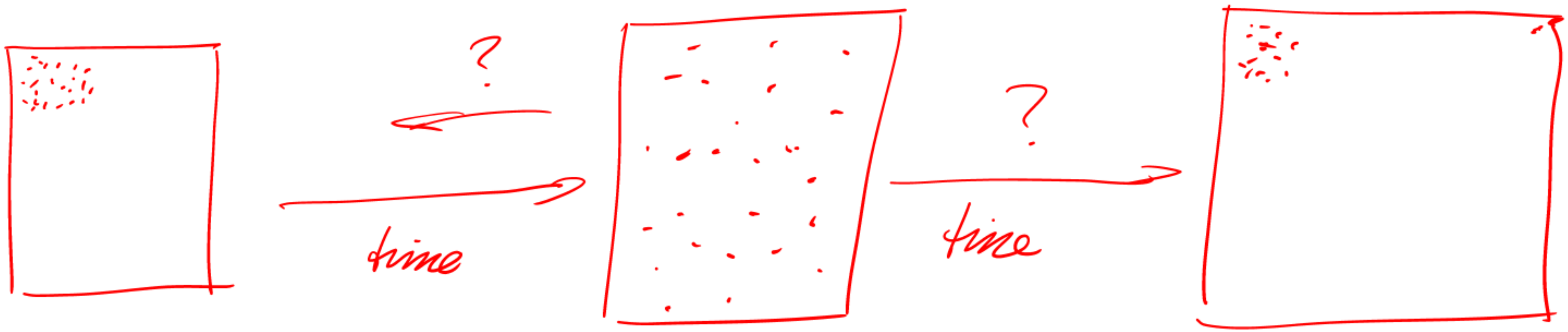
Loschmidt Paradox (1876)

If microscopic laws are time reversible, entropy will decrease if initial state is taken with all velocities reversed. start with "later state" equally likely if $\vec{p}_j \to -\vec{p}_j$

Poincare (1890) and Zermelo (1896)

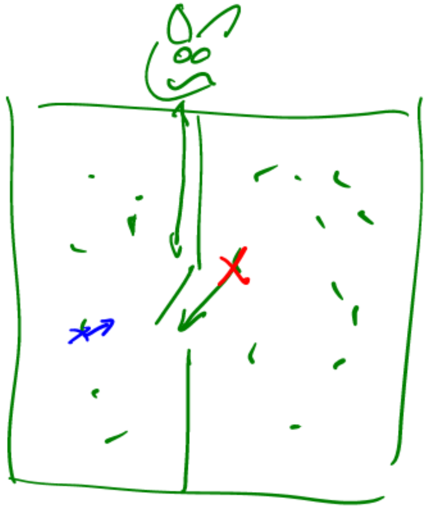
Recurrance theorem

In certain dynamical systems the time evolved state will return arbitrarily close to the initial state in finite time.



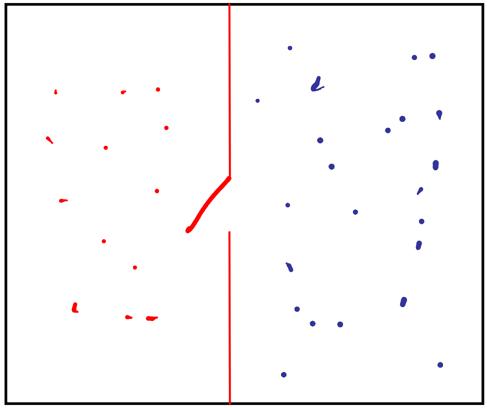
$$|\Gamma_{initial} - \Gamma_{final}| < \delta\Gamma \qquad t(\delta\Gamma)$$

Maxwell's Demon (1867)

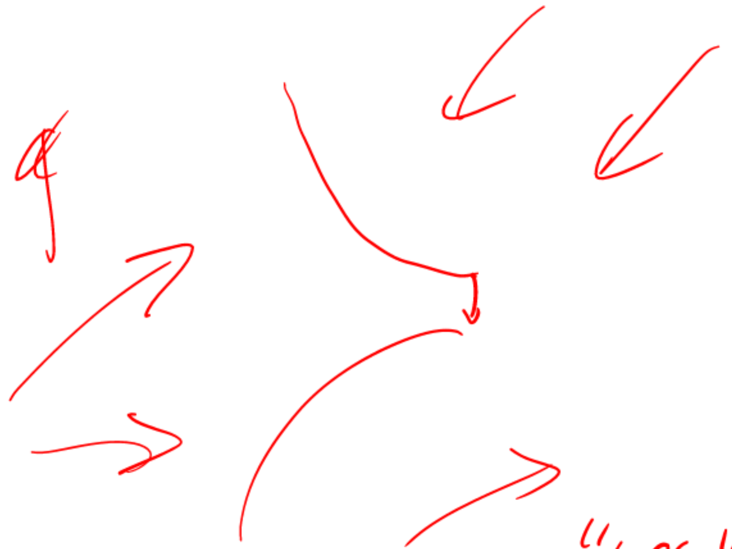


$T_A = T_B$

time



$T_A > T_B$



open if cold particle from left
or hot from right
closed otherwise

* no asymmetric passive "filter"
should be possible

Maxwell Demon requires negative entropy
to work

— measure momenta of particle
→ information increase
→ entropy decrease

— blocking ?