

# Mapping as a probe for heating suppression in periodically driven quantum many-body systems

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## 1 Abstract

2 Experiments on periodically driven quantum systems have effectively realized  
 3 quasi-Hamiltonians, in the sense of Floquet theory, that are otherwise inaccessi-  
 4 ble in static condensed matter systems. Although the Floquet quasi-Hamiltonians  
 5 are time-independent, however, these continuously driven systems can still suf-  
 6 fer from heating due to a secular growth in the expectation value of the time-  
 7 dependent physical Hamiltonian. Here we use an exact space-time mapping to  
 8 construct a class of many-body systems with rapid periodic driving which we  
 9 nonetheless prove to be completely free of heating, by mapping them exactly onto  
 10 time-independent systems. The absence of heating despite the periodic driving  
 11 occurs in these cases of harmonically trapped dilute Bose gas because the driving  
 12 is a certain periodic but anharmonic modulation of the gas's two-body contact  
 13 interaction, at a particular frequency. Although we prove that the absence of  
 14 heating is exact within full quantum many-body theory, we then use mean-field  
 15 theory to simulate 'Floquet heating spectroscopy' and compute the heating rate  
 16 when the driving frequency is varied away from the critical value for zero heating.  
 17 In both weakly and strongly non-linear regimes, the heating rate as a function of  
 18 driving frequency appears to show a number of Fano resonances, suggesting that  
 19 the exactly proven absence of heating at the critical frequency may be explained  
 20 in terms of destructive interferences between excitation modes.

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## 46 1 Introduction

47 If a quantum Hamiltonian depends on time periodically, then the system possesses a discrete  
 48 time translation symmetry, analogous to the discrete spatial translation symmetry of a lattice  
 49 potential. Analogous to Bloch waves, the periodically driven system allows a complete set  
 50 of solutions to the time-dependent Schrödinger equation, which have the form of a quasi-  
 51 energy phase factor times a time-periodic wave function. The Fourier series components of  
 52 the periodic wave function obey time-independent Schrödinger equations, and in this sense  
 53 periodic driving can effectively realize new time-independent Hamiltonians [1–3]. The growing  
 54 subject of *Floquet engineering* [4–10] seeks to exploit this possibility to simulate exotic many-  
 55 body dynamics that is not found in static condensed matter systems [11–13], in order to  
 56 answer fundamental questions or develop technological applications. Floquet engineering is  
 57 now a widespread tool in the realm of ultracold quantum gases [14–20]. Among many other  
 58 utilizations, it may allow achieving the Mott-insulator-to-superfluid transition of two-species  
 59 hardcore bosons [21].

60 The initial states that can actually be prepared in a driven system, however, may be limited  
 61 by the actual time-dependent Hamiltonian rather than by the corresponding Floquet effective  
 62 Hamiltonian, because it is still the time-dependent Hamiltonian which actually determines  
 63 the system’s time evolution. Measurable observables likewise evolve under the actual time-  
 64 dependent Hamiltonian. Even though the Floquet effective Hamiltonian is time-independent,  
 65 therefore, it is a generic problem for Floquet engineering that a continuously driven system  
 66 typically suffers from heating [20, 22–26]. For initial quantum states that can be prepared in

67 experiments, the physical energy of the system, *i.e.* the expectation value of the actual time-  
68 dependent Hamiltonian, may not merely be periodic and bounded, but may grow secularly  
69 over long times. This long-term heating may mask the more interesting phenomena which  
70 are the target of Floquet engineering.

71 In a Fermi-Hubbard system [27], deviations from the expected behavior in the effective  
72 Hamiltonian may clearly arise for long modulation times when heating processes dominate.  
73 For quantum critical systems described at low energy by a conformal field theory, multiple  
74 dynamical regimes clearly occur depending on the drive frequency. For slow driving and long  
75 times, the system becomes unstable and heats up to infinite temperature. In the limit where  
76 the driving frequency is much faster than any natural frequencies of the problem, one can use  
77 the Magnus expansion to find an appropriate Floquet Hamiltonian which is robust against  
78 heating [3]. In some cases interference effects have also been shown to limit heating [25, 28].  
79 Further examples showing ways to avoid heating by periodic driving are, however, of interest.

80 One technique, which has yielded exact results in many-body theory, is the use of space-  
81 time mappings to relate non-trivial systems to simpler ones. This approach has been ap-  
82 plied to quantum gases in various special regimes of inter-particle interaction or dimen-  
83 sions [29–31]. In these cases the mappings have been performed by constructing exact non-  
84 trivial time-dependent many-body wave functions from simpler wave functions that were  
85 previously known, by appropriately transforming space and time coordinates. In general the  
86 previous use of space-time mappings in many-body theory has been restricted to looking for  
87 exact solutions in special cases. In a recent work, however, we showed that beyond the use  
88 of space-time mappings for exact solution, a much more general kind of exact mapping turns  
89 out to be possible between pairs of many-body time evolutions [32]. Even though the ex-  
90 act solutions may no longer be available for either evolution, the mapping between the two  
91 potentially very different evolutions remains exact. And in Ref [33], it was shown that that  
92 this intriguing mapping between many-body quantum systems can even be extended to open  
93 systems.

94 In the present paper, we apply our mapping to address the heating problem of Floquet  
95 engineering in quantum many-body systems. The structure of the paper is as follows: Sec-  
96 tion 2 reviews the quantum field mapping scheme for dilute quantum gases and uses it to  
97 construct a class of periodic modulations of the interaction strength, which must have exactly  
98 zero heating, because they can be mapped onto a time-independent system. Such a class is  
99 discussed in section 3 with a detailed example, where the evolution of a quantum gas with  
100 periodically driven interactions in a static harmonic trap is mapped onto the evolution of the  
101 gas with un-driven interaction in a (different) static harmonic trap.

102 Our main results begin in section 4, where we shift attention away from the exact mapping  
103 to investigate *why* heating vanishes in this special case, by examining a larger class of peri-  
104 odic modulations which includes our special zero-heating case, but should otherwise exhibit  
105 heating. In particular we consider a quasi-one-dimensional Bose gas in a harmonic trap, with  
106 contact interactions of periodically modulated strength; since exact solutions of the full quan-  
107 tum problem are unavailable, we fall back on Gross-Pitaevskii mean-field theory. Mean-field  
108 theory reproduces the exact quantum many-body result of zero heating at a critical modula-  
109 tion frequency (double the trap frequency), but also allows us to compute heating rates for  
110 a range of different driving frequencies, in different regimes of both driving amplitude and  
111 interaction strength. We find Fano-like resonances in the heating rate, suggesting a generic  
112 mechanism for heating suppression in Floquet systems.

113 In Section 5 we then apply our mapping to the cases with heating, mapping experiments

114 with modulated interactions in static traps onto experiments with constant interactions in  
 115 modulated traps. Here our results are more cautionary: plausible arguments based on the  
 116 modulated-trap version of the experiments may predict dramatic ‘Bose fireworks’ heating in  
 117 cases where it does *not* in fact occur, at least in mean-field theory. Section 6 concludes the  
 118 work and draws some perspectives.

## 119 2 Mapping and driving

120 In this section we review the exact mapping identities of [32] and [33] and use them to construct  
 121 a rapidly driven system which has zero heating because it is only a spacetime transformation  
 122 of an undriven system. Although the mapping is also applicable for general two-particle  
 123 interactions [32], we focus here on systems of dilute Bose gas with contact interactions.

### 124 2.1 Mapping identities

125 Consider a quantum gas in  $D$  dimensions with particles of mass  $M$  subject to contact two-  
 126 body interactions. Evolution of the gas is described in the Heisenberg picture of quantum  
 127 dynamics by a time-dependent quantum field operator  $\hat{\psi}(\mathbf{r}, t)$  that satisfies the Heisenberg  
 128 equation of motion

$$i\hbar \frac{\partial}{\partial t} \hat{\psi} = -\frac{\hbar^2}{2M} \nabla^2 \hat{\psi} + V(\mathbf{r}, t) \hat{\psi} + g(t) \hat{\psi}^\dagger \hat{\psi}^2, \quad (1)$$

129 where  $V(\mathbf{r}, t)$  denotes the trapping potential, and  $g(t)$  stands for the two-particle interaction  
 130 strength. Highly controllable time-dependent interactions are routinely achieved in current  
 131 quantum gas laboratories, for example via Feshbach resonance management [34–38]. Ex-  
 132 periments with time-dependent interactions are currently of high interest in investigating  
 133 non-equilibrium many-body evolutions [39–41].

134 Our spacetime mapping identities is the following. If  $\hat{\psi}_A(\mathbf{r}, t)$  is a solution to (1) for  
 135 potential  $V = V_A(\mathbf{r}, t)$  and interaction strength  $g(t) = g_A(t)$ , then the following  $\hat{\psi}_B(\mathbf{r}, t)$  is a  
 136 solution for the following  $V = V_B$  and  $g = g_B$ :

$$\begin{aligned} \hat{\psi}_B(\mathbf{r}, t) &= e^{-\frac{iM}{2\hbar} \frac{\dot{\lambda}}{\lambda} r^2} \lambda^{D/2} \hat{\psi}_A(\lambda \mathbf{r}, \tau(t)) \\ V_B(\mathbf{r}, t) &= \lambda^2 V_A(\lambda \mathbf{r}, \tau(t)) + \frac{Mr^2}{2} \lambda^3 \left( \frac{1}{\lambda^2} \frac{d}{dt} \right)^2 \lambda \\ g_A(t) &\mapsto g_B(t) = \lambda(t)^{2-D} g_A(\tau(t)) \\ \tau(t) &= \int_0^t \lambda(t')^2 dt', \end{aligned} \quad (2)$$

137 where  $\lambda = \lambda(t)$  is an arbitrary function subject only to the constraints  $\lambda(0) = 1$ ,  $\dot{\lambda}(0) = 0$ , for  
 138  $\dot{\lambda}(t) \equiv d\lambda/dt$ . If we impose the Heisenberg-picture initial condition  $\psi_A(\mathbf{r}, 0) = \psi_B(\mathbf{r}, 0)$  then  
 139 the two time-dependent field operators  $\hat{\psi}_{A,B}$  describe two different experiments on a dilute  
 140 Bose gas prepared in the same initial state. Since  $V_A$  and  $V_B$  as well as  $g_A$  and  $g_B$  can easily  
 141 be quite different, the A and B experiments can involve very different manipulations of the  
 142 gas sample. Nonetheless the two second-quantized destruction fields are exactly related by  
 143 this simple mapping, which involves a time- and space-dependent phase factor and a rescaling  
 144 of space, and which relates the two experiments at different times, such that  $t_A = \tau(t_B)$ .

145 Any possible experimental observables can be represented as expectation values of  $N$ -point  
 146 functions of the second-quantized field operators,

$$F_{\text{ex}}(\mathbf{R}, \mathbf{R}', t) = \left\langle \prod_{j=1}^N \hat{\psi}_{\text{ex}}^\dagger(\mathbf{r}'_j, t) \prod_{j=1}^N \hat{\psi}_{\text{ex}}(\mathbf{r}_j, t) \right\rangle, \quad (3)$$

147 where the subscript ex refers to any of the experiments A and B. The mapping between the  
 148 quantum fields relates the  $N$ -point functions to each other as follows:

$$F_B(\mathbf{R}, \mathbf{R}', t) = \lambda^{ND} e^{-\frac{iM}{2\hbar} \frac{\dot{\lambda}}{\lambda} \sum_{j=1}^N (r_j^2 - r_j'^2)} F_A(\lambda \mathbf{R}, \lambda \mathbf{R}', \tau(t)), \quad (4)$$

149 where  $t \equiv t_B$ ,  $\tau(t) \equiv t_A$ . Thus the mapping truly implies that either of the two experiments  
 150 is a perfect analog simulation of the other one, with any measurements at any times in one  
 151 experiment corresponding, according to (4), to measurements at corresponding (different!)  
 152 times in the other experiment. The mapping identities for field operators, trapping potentials,  
 153 interaction strengths and  $N$ -point functions hold for any initial state of the system, pure or  
 154 mixed and no matter how far from equilibrium it is, as long as the initial state is the same in  
 155 both experiments A and B.

156 One practical application of the mapping, as indicated in [32], is to use it to simulate  
 157 a more difficult experiment B exactly by mapping to it from a technically more feasible  
 158 experiment A. An example given in [32] was a mapping between an A in which the harmonic  
 159 trap is simply turned off (a ballistic expansion experiment) and a B in which the contact  
 160 interaction strength is ramped to infinity. The mapping is valid, however, for *arbitrary*  $\lambda(t)$ .  
 161 With periodic  $\lambda(t)$ , therefore, one can effectively achieve more complex periodically driven  
 162 experiments, for instance with periodic modulation of the interaction strength, by performing  
 163 only simpler ones, in which for example only the trapping potential is varied.

164 In this paper we will begin with the most trivial limit of this application: the effective  
 165 realization of a periodically driven experiment B from a *time-independent* experiment A. The  
 166 point of this especially simple mapping is not just that a time-independent experiment is  
 167 easier than a time-dependent one: it is that in a time-independent experiment there can be  
 168 no secular heating, and so therefore any experiment which can be mapped exactly onto a  
 169 time-independent one according to (2) must also avoid secular heating, even if it includes  
 170 driving.

## 171 2.2 Driving without heating

172 Any particular mapping between two experiments A and B is defined by the arbitrary function  
 173  $\lambda(t)$  of Eqs. (2). A concrete example, which as we will see will map an undriven evolution in  
 174 A onto an experiment B with periodic driving in the contact interaction strength  $g_B(t)$ , is

$$\lambda(t) = \frac{1}{\sqrt{\frac{1-\gamma^2}{2} \cos(2\omega t) + \frac{1+\gamma^2}{2}}}, \quad (5)$$

175 where  $\omega$  and  $\gamma$  are arbitrary constants, taken as positive without loss of generality.

176 This  $\lambda(t)$  is periodic in time, with  $\lambda(n\pi/\omega) = 1$ ,  $\dot{\lambda}(n\pi/\omega) = 0$  for all integer  $n$ . The  
 177 general mapping (2) thus implies that

$$\hat{\psi}_A(\mathbf{r}, \tau(\frac{n\pi}{\omega})) = \hat{\psi}_B(\mathbf{r}, \frac{n\pi}{\omega}), \quad (6)$$

178 so that all possible observables in experiment A at times  $t_A = \tau(n\pi/\omega) = \gamma^{-1}n\pi/\omega$  will  
 179 exactly coincide with those in experiment B at times  $t_B = n\pi/\omega$ . If there is no secular  
 180 heating in experiment A, therefore, there cannot be any secular heating in experiment B.

181 To ensure that there is no secular heating in experiment A, we simply choose

$$V_A = \gamma^2 \frac{M\omega^2 |\mathbf{r}|^2}{2} \quad (7)$$

182 for the same arbitrary  $\gamma$  and  $\omega$  that appear in  $\lambda(t)$ , and select any *time-independent* contact  
 183 interaction strength  $g_A = g_0$ . This makes the Hamiltonian for the gas in experiment A  
 184 completely time-independent. The mapping (2), however, yields

$$V_B = \frac{M\omega^2 |\mathbf{r}|^2}{2} \quad (8)$$

$$g_B(t) = g_A \lambda(t)^{2-D}. \quad (9)$$

185 Experiment B thus also has a static harmonic trap with frequency  $\omega_B = \omega$ , generally different  
 186 (since  $\gamma$  can be anything) from the trap frequency  $\omega_A = \gamma\omega$  in experiment A. In experiment  
 187 B, however, the contact interaction strength  $g_B(t)$  is time-dependent whenever  $\gamma \neq 1$  and the  
 188 effective dimensionality of the trapped gas is  $D \neq 2$ .

189 In particular  $g_B(t)$  is anharmonically modulated (except for the degenerate case  $\gamma = 1$ )  
 190 with frequency  $2\omega = 2\omega_B$  and with an amplitude that depends on  $\gamma$ , as illustrated for the  
 191 case  $\gamma = 1.5$  in Fig. 1. For a quasi-1D Bose gas ( $D = 1$ ) we have simply  $g_B(t) = g_0 \lambda(t)$   
 192 when  $g_A = g_0$  is constant; the time average of the interaction strength felt by the atoms in  
 193 experiment B is

$$\langle g_B \rangle = \frac{\omega_B}{\pi} \int_0^{\pi/\omega_B} g_A \lambda(t) dt = \frac{2}{\pi} g_A K(1 - \gamma^2), \quad (10)$$

194 where  $K$  denotes the complete elliptic integral of the first kind. The interaction strength  
 195  $g_B(t)$  oscillates in time around  $\langle g_B \rangle$ , as in Fig. 1(b). This specific time dependence of  $g_B(t)$   
 196 is naturally an experimental challenge to realize precisely but the experimental technology to  
 197 achieve it for trapped ultracold gases certainly exists.

198 In spite of this possibly (depending on  $\gamma$ ) strong modulation of  $g_B(t)$ , however, the exact  
 199 quantum field mapping of (2) ensures that all observables in experiments A and B are always  
 200 related, at the different times  $t_B = t$  and  $t_A = \tau(t)$ , by the simple scaling relation (4), which  
 201 in particular reduces to identity after every driving period. If the shared initial state of the  
 202 two experiments is time-independent in A, then the time-dependent state in B will simply  
 203 oscillate forever periodically. Regardless of the initial state, the evolution in A will obviously  
 204 conserve energy, and since the mapping between the two systems is periodic, there can never  
 205 be any secular growth in the energy in B.

206 The particular form of  $\lambda(t)$  chosen in (5) is a convenient example because according to (2)  
 207 it yields a time-independent  $V_B$ , so that only the interaction is modulated in experiment B.  
 208 With a generic periodic  $\lambda(t)$  the time-independent experiment A would be mapped onto a class  
 209 of B experiments with arbitrary periodic driving in  $g_B(t)$ , but with a simultaneous modulation  
 210 of  $V_B(t)$  that has to be synchronized non-trivially with  $g_B(t)$ , in accordance with (2). Our  
 211 mapped B experiments with exactly no heating are thus always quite special cases of periodic  
 212 driving; we continue with (5) and its static  $V_B$  for the rest of this paper simply because the  
 213 cases with time-dependent  $V_B$  are equally special and more complicated to describe.

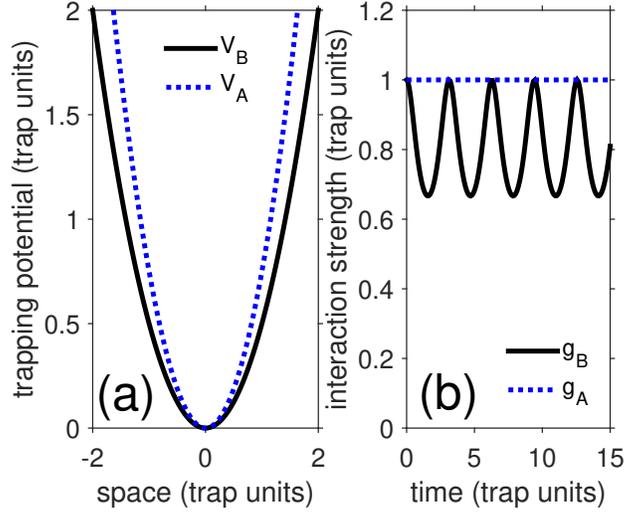


Figure 1: Sketch of (a) trapping potentials  $V_B = M\omega_B^2 x^2/2$  and  $V_A = M\gamma^2\omega_B^2 x^2/2$  over space  $x$ , and (b) interaction strengths in both experiments A and B over time  $t$  according to Eq. (8). The time is measured in the trap units of Experiment B. Experiment A is a static evolution as both the trap and interaction strengths are kept constant in time. In experiment B, while the harmonic trap is static, the scattering length is periodically driven in time in such a way that the interaction strength is modulated with driving frequency  $2\omega_B$ . We used  $\gamma = 1.5$  and  $g_A = 1$ , which yields  $\langle g_B \rangle \approx 0.8$ .

### 214 2.3 Mapping of times

215 We will describe our evolutions in the time  $t = t_B$  of experiment B, but it is straightforward  
 216 to derive the corresponding time in experiment A. From Eqs. (2), we find

$$t_A = \frac{\tan^{-1}[\gamma \tan(\omega_B t_B)] + n_B \pi}{\gamma \omega_B}, \quad (11)$$

217 where  $n_B = \lfloor \frac{2\omega_B t_B + \pi}{2\pi} \rfloor$ , with  $\lfloor \dots \rfloor$  denoting the floor function. Inversely, then we also have

$$t_B = \frac{\arctan[\gamma^{-1} \tan(\omega_A t_A)] + n_A \pi}{\gamma^{-1} \omega_A}, \quad (12)$$

218 where  $n_A = \lfloor \frac{2\omega_A t_A + \pi}{2\pi} \rfloor$ . Inserting (12) into (5), we can express the factor  $\lambda(t)$  in terms of the  
 219 time in experiment B as  $\lambda(t(\tau)) =: \tilde{\lambda}(\tau) \equiv \tilde{\lambda}(t_A)$

$$\tilde{\lambda}(t_A) = \sqrt{\frac{1 - \gamma^{-2}}{2} \cos(2\omega_A t_A) + \frac{1 + \gamma^{-2}}{2}}. \quad (13)$$

220 The reciprocal relationship between  $\lambda$  in (5) and  $\tilde{\lambda}$  in (13) is generic for the spacetime mapping  
 221 (2): the inverse mapping from B back to A is always simply the mapping with  $\lambda \rightarrow 1/\lambda$  and  
 222  $t_A$  and  $t_B$  exchanged.

## 2.4 Why the absence of heating?

Our mapping has thus already shown the existence of a class of special cases of periodically driven quantum many-body systems with exactly no secular heating. Our further goal in this paper is to shed light on the mechanism by which these special cases avoid heating, since this mechanism will likely operate to some degree in a much broader range of cases of driving and is therefore of general interest. Since we cannot actually solve the full quantum many body problem, however, we will proceed to investigate dynamical mechanisms for avoidance of heating within Gross-Pitaevskii mean-field theory for the quasi-one-dimensional (quasi-)condensed Bose gas with weak contact interactions. It is straightforward to show [32] that the mapping (2), which is exact in the Heisenberg picture of the full quantum theory, is also valid in the corresponding mean-field theory.

## 3 Mapping and driving in mean-field theory

We illustrate concretely how the mapping between the time-independent and periodically driven experiments works within 1D mean-field theory for condensed bosons.

### 3.1 Numerical experiments with a pair of evolutions

We consider as a sample quantum gas a cigar-shaped (to the point of being quasi-one-dimensional) Bose-Einstein condensate that is described with a  $c$ -number field  $\psi(x, t)$ , the condensate wave function, governed by a Gross-Pitaevskii (GP) equation which is the mean-field counterpart of the Heisenberg equation (1),

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} \psi + \frac{M[\omega(t)]^2 x^2}{2} \psi + g(t) |\psi|^2 \psi. \quad (14)$$

The interparticle interaction is constant and the gas is confined in a harmonic trap with trap frequency  $\omega(t)$  as sketched in Fig. 1(a). In order to solve the GP equation numerically, we prepare the initial state within the Thomas-Fermi regime using imaginary time relaxation. The same initial state will be used for both experiments A and B.

In experiment A the un-driven system will simply remain in its initial ground state forever, while in experiment B the interaction strength is periodically modulated according to Eq. (8); as one could anticipate the gas density profile will not remain constant in B, since in B the Hamiltonian is periodically time-dependent. The implication of our mapping, however, is that all that will happen in B is a collective breathing mode, of which the amplitude will remain constant forever with zero secular growth. Without invoking our mapping, but simply numerically solving the Gross-Pitaevskii equation for B with the initial state as in A, we indeed obtain just such a breathing: see Fig. 2(b).

### 3.2 Mapping the two numerical experiments

We now directly confirm that experiments A and B as shown in Fig. 2 are mapped onto each other by (2) with the operator fields  $\hat{\psi}_{A,B}$  replaced by the  $c$ -number order parameters  $\psi_{A,B}$ . Figure 3 shows the correspondingly obtained densities  $|\psi_B(x, t)|_{\text{map}}^2$  and  $|\psi_A(x, t)|_{\text{map}}^2$  obtained by mapping the densities  $|\psi_A(x, t)|^2$  and  $|\psi_B(x, t)|^2$  given by the Gross-Pitaevskii evolution that were displayed in panels (a) and (b) of Fig. 2. Comparing Figs. 2 and 3, it is

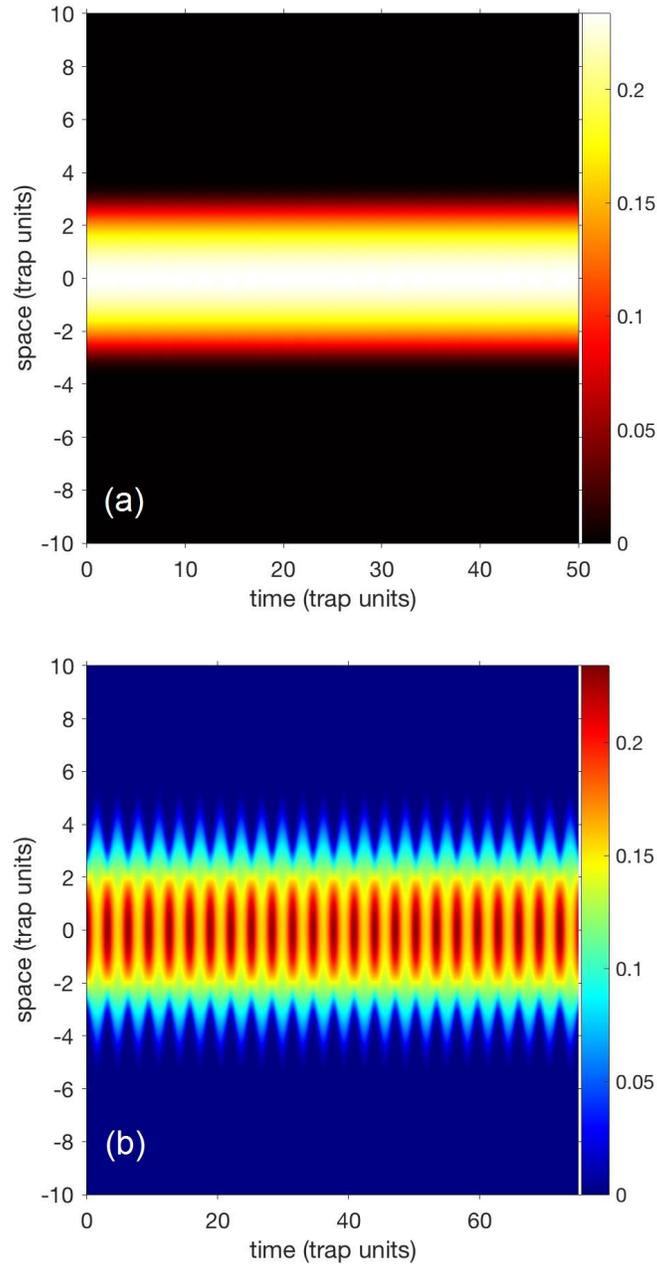


Figure 2: Density evolution in space and time  $|\psi(x, t)|^2$  in two different experiments, namely (a) a static problem A and, (b) a Floquet problem B. In experiment A, we used the trapping frequency  $\omega_A = 1.5\omega$  and the interaction strength  $g_A = g_0 = 1.0$ . In experiment B, the trapping frequency is  $\omega_B = \omega$  and the interaction strength  $g_B(t) = g_0\lambda(t)$  for  $\lambda(t)$  given by (5) with  $\gamma = 1.5$ . The time is measured in units of  $1/\omega$  and position in units of the corresponding trap length.

260 impossible to tell that the plots have not just been swapped for each other. The mapping is  
 261 exact.

262 The mapping is however not trivial. The space and time axes in both plots, along with the  
 263 density scales, have been transformed according to (2) and its inverse. Note in particular the  
 264 difference of the time spans in panels (a) and (b). At the end of the displayed experiments,  
 265 we have  $t_A \equiv 50$  and  $t_B \equiv 75$  in the same trap-B-based natural units, as is readily obtained  
 266 from Eq. (11).

### 267 3.3 How is heating avoided?

268 The complete absence of heating in this special one-parameter family of periodic driving exper-  
 269 iments, with the interaction strength modulated at exactly twice the static trap frequency  
 270 and with a very particular  $\gamma$ -dependent anharmonic time dependence, makes this special case  
 271 interesting. It shows by example that secular heating can be avoided. In itself it is a mere  
 272 curiosity, though. The more generally interesting phenomenon which this special case may re-  
 273 veal is the dynamical mechanism of heating avoidance, since this mechanism can be expected  
 274 to operate, with greater or lesser effect, in a wide range of cases.

275 We therefore expand our attention now to a wider range of periodically driven experiments,  
 276 beyond those which lack heating because they map onto undriven experiments. In particular  
 277 we consider experiments of the same form as our previous experiment B for  $D = 1$  as above,  
 278 and in which the temporal modulation of  $g(t)$  has the same anharmonic form (5,8) as in our  
 279 mapped B experiments, but now with an arbitrary driving frequency:

$$g(t) = g_0 \left( \frac{1 + \gamma^2}{2} + \frac{1 - \gamma^2}{2} \cos(\nu\omega t) \right)^{-1/2} \quad (15)$$

280 for arbitrary real  $\gamma$ ,  $g_0$ , and  $\nu$ . Our mapping results so far show that there will be no heating  
 281 for  $\nu = 2$ . What happens away from  $\nu = 2$ ?

## 282 4 Heating rate and suppression

283 In this our main section we first face the basic question of how driving-induced heating can be  
 284 quantified from numerical computations within mean-field theory. We show how the heating  
 285 rate can be computed numerically. We then employ this method to see how heating rate in  
 286 our driven 1D (quasi-)condensate varies with driving frequency  $\nu\omega$  and interaction strength  $g$ .  
 287 We will find that the heating rate shows troughs and peaks of a particular form that suggests,  
 288 by analogy with other dynamical systems, that heating avoidance occurs through destructive  
 289 interference of competing collective modes.

### 290 4.1 Numerical method for estimating heating

291 The secular heating rate is defined here as the average rate of change of the instantaneous  
 292 energy of the system at long times; we express the heating rate dimensionlessly in terms of  
 293 trap units  $\hbar\omega^2$ . In order to compute this heating rate, we first compute the instantaneous  
 294 energy and then determine its long-term average. As a trivial simplification we subtract the  
 295 initial energy and compute  $\Delta E(t) = E(t) - E(0)$ .

296 Within Gross-Pitaevskii mean-field theory for the (quasi-)condensed 1D Bose gas, the

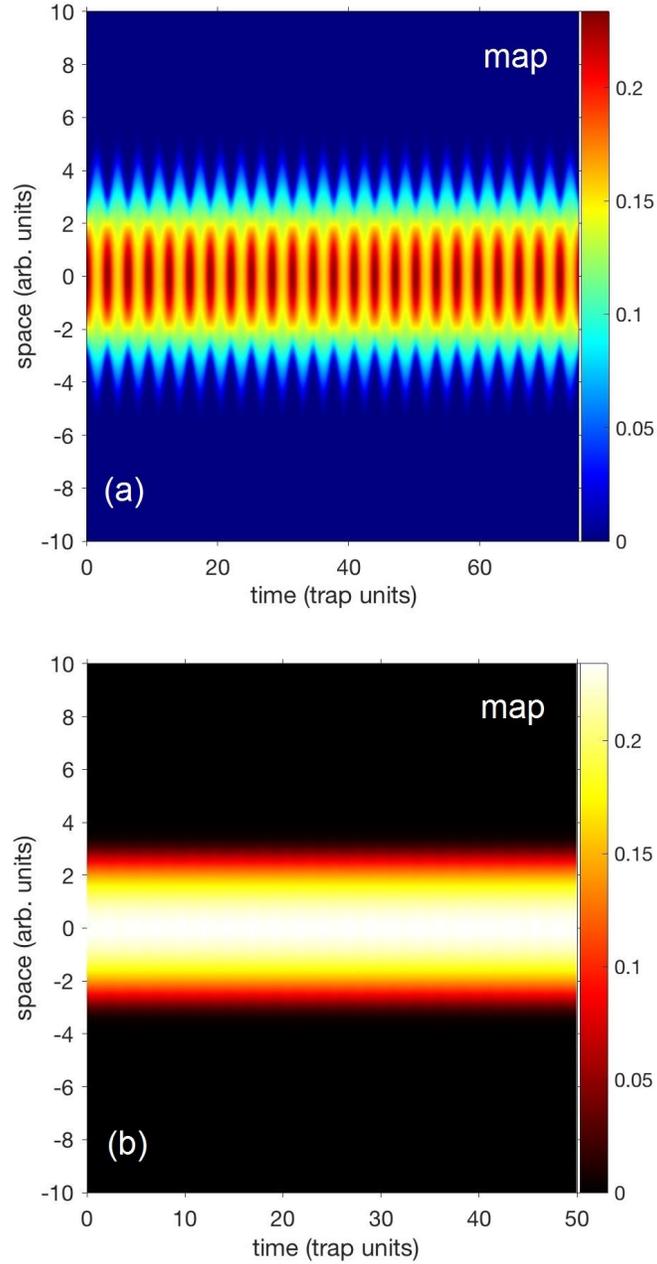


Figure 3: Space-time evolution of the densities (a)  $|\psi_B(x,t)|_{\text{map}}^2$  and (b)  $|\psi_A(x,t)|_{\text{map}}^2$ , obtained by mapping the densities obtained in experiments A and B, respectively, using the relation (2) and its inverse. There are no differences between these plots and those of Fig. 2, confirming that our quantum field mapping is also exact in mean-field theory.

297 instantaneous energy is

$$E(t) \rightarrow E_{\text{GP}}(t) = \int dx \left[ \frac{\hbar^2}{2M} \left| \frac{\partial \psi}{\partial x} \right|^2 + V(x) |\psi|^2 + \frac{g(t)}{2} |\psi|^4 \right]. \quad (16)$$

298 The energy difference  $\Delta E(t)$  is in general not constant and may have complicated temporal  
 299 behavior. It exhibits multiple time scales, including a driving period and a beat period, as  
 300 well as some longer time scales. At large enough times, however, we find numerically that  
 301  $\Delta E(t)$  becomes dominated by a linear growth with a well-defined slope. We identify this slope  
 302 as the heating rate.

303 It is straightforward to detect the emergence of the linear energy growth because it con-  
 304 tinues steadily until it dominates clearly. We therefore simply evolve numerically under the  
 305 Gross-Pitaevskii nonlinear Schrödinger equation (14) over a total time  $\tau$  (many trap periods),  
 306 recording the energy  $\Delta E(t)$  at a discrete set of evenly spaced  $t_n$  which are all whole-number  
 307 multiples of the driving period  $2\pi/(\nu\omega)$ . On this sequence of  $\Delta E(t_n)$  we then perform a linear  
 308 regression analysis, fitting it to the linear model

$$\Delta E(t) \approx \mathcal{E} + \beta_\tau t, \quad \text{with } t \in [0, t_{\max}]. \quad (17)$$

309 See Fig. 4(a). While the intercept fitting parameter  $\mathcal{E}$  is of no particular importance in our  
 310 problem, the slope or gradient fitting parameter  $\beta_\tau$  represents the secular power gain rate of  
 311 the system, due to the driving, over the time scale  $\tau$ . By comparing  $\beta_\tau$  for different large  
 312 values of  $\tau$  (up to thousands of trap periods) we find that although there is an initial transient  
 313 regime in which  $\beta_\tau$  varies significantly with  $\tau$ , at large enough  $\tau$  the heating rate approaches  
 314 a constant (see (Fig. 4(b)), which we then identify as the heating rate  $\Gamma := \beta_\infty$ .

## 315 4.2 Heating rate for different interaction strengths

316 Thanks to sophisticated Feshbach techniques available in present-day quantum gas laborato-  
 317 ries, numerous experiments have been achieved with variable interaction strengths [34–38].  
 318 We therefore pause briefly here to investigate how the heating of our 1D mean-field gas is  
 319 affected by the interaction strength prefactor  $g_0$ . This serves as a generic check on our method  
 320 for determining heating; we must expect that heating is generally weaker for weakly inter-  
 321 acting systems that are periodically driven, since driven harmonic oscillators reach constant-  
 322 amplitude steady states, except exactly on resonance.

323 Since the only driving in our system is in the interaction we can have no heating at all for  
 324  $g_0 \rightarrow 0$ , but we can confirm the reasonable behavior of our numerical heating rates by seeing  
 325 how they tend to increase with  $g_0$ , as shown in Fig. 5. Globally, the heating rate increases  
 326 with the interaction strength for all driving frequencies  $\nu\omega$ . The average rate of heating rate  
 327 increase with  $g_0$  itself increases with the driving frequency. In agreement with the results in  
 328 our earlier sections, the heating rate remains zero for all  $g_0$  in the special non-heating case  
 329  $\nu = 2$  that can be mapped onto the time-independent system by the space-time rescaling (2).  
 330

## 331 4.3 Heating rate for different driving frequencies: heating avoidance and 332 hidden adiabaticity

333 We now proceed to consider the effect on heating of the driving frequency  $\nu\omega$ ; this is the core of  
 334 our paper, and the heating spectrum shown in Fig. 6 is our main numerical result. Our results  
 335 shown for  $t_{\max} = 2500$  are not discernibly different from those with  $t_{\max} = 1125$ , confirming  
 336 that we are analyzing the asymptotic long-time regime of secular heating. For  $\nu = 0$  heating  
 337 must vanish exactly, since the system is static, and for sufficiently low  $\nu$  the system should still  
 338 avoid heating, because it should adapt to the slowly modulated Hamiltonian adiabatically.

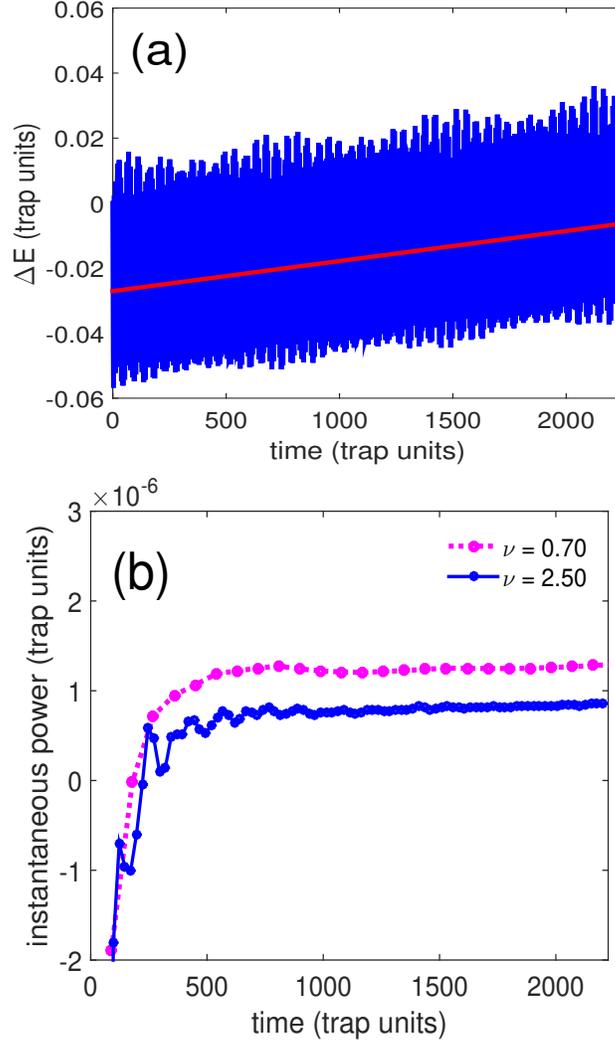


Figure 4: (a) Time evolution of the energy difference  $\Delta E$  along with the fitting model (red straight line). (b) Instantaneous power  $\beta_\tau$  received by the gas from the drive for two different driving frequencies  $\nu = 0.7$  and  $2.5$ ; we used the interaction strength in the form (8) with  $g_0 = 1.0$  and modulation amplitude parameter  $\gamma = 1.5$ . The power is measured in trap units of  $\hbar\omega^2$ . The time when the power stops changing considerably allows us to determine the long-time regime, which is roughly  $t \in [600, \infty[$  and  $[1000, \infty[$  for the frequencies  $\nu = 0.7$  and  $2.5$ , respectively. To obtain the power  $\beta_\tau$ , we considered  $\Delta E(t)$  at times  $t = 2(n+n_0)\pi/(\nu\omega) \equiv t_n$ , for  $n = 0, 1, \dots, N$  and  $t_N \leq t_{\max}$ . Then we obtained  $\beta_\tau$  through a linear fit of the data in the set  $\{\Delta E(t_0), \dots, \Delta E(t_N)\}$  for  $\tau \in \{t_1, \dots, t_N\}$ . We used  $n_0 = 4$ .

339 We do not see this in Fig. 6, however, because in fact the lowest  $\nu$  that we computed was  
 340  $\nu = 0.05$ , which is evidently not slow enough for the system to remain adiabatic over very  
 341 long times. For  $\nu < 2$  we do see the heating increasing slightly with drive frequency, as we  
 342 noted in Fig. 5 above. Over the extended range  $2 < \nu < 10$ , however, this increasing trend  
 343 does not continue and instead the background heating rate remains nearly constant. Heating  
 344 should again decrease trivially at very high frequencies, as the system responds only to the

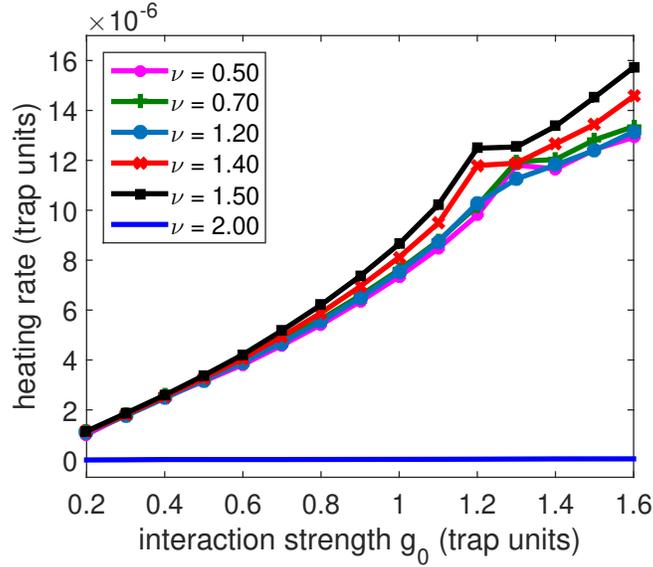


Figure 5: Heating rate as a function of interaction strength at small driving frequencies and at the heating trough frequency. The frequencies are given by  $\nu = 0.5, 0.7, 1.20, 1.40, 1.5$  (less than the sharp resonance frequency  $\nu \approx 1.9$ ) and 2.0 (heating zero); we used the runtime  $t_{\max} = 1125$  and driving strength parameter  $\gamma = 1.5$ .

static time-averaged potential, but our one-dimensional Gross-Pitaevskii system has many high-frequency collective modes, and it is a nonlinear system with finite-amplitude driving; the time-averaged high-frequency limit is clearly well above  $\nu = 10$ .

Against the essentially flat background heating in Fig. 6, two dramatic features are seen: sharp dips and spikes in the heating rate at a number of particular driving frequencies.

**Heating spikes.** Many heating spikes can be seen in the heating spectrum. They appear close to frequencies that are associated with parametric resonances [42–45]. In general parametric resonance can be excited whenever the driving frequency  $\nu\omega = 2\Omega_n/m$ , where  $\Omega_n$  is an eigenfrequency of the system and  $m$  any positive integer.

For vanishing interactions the eigenfrequencies in our dimensionless trap units are  $\Omega_n = n\omega$ , with  $n$  being any positive integer. With  $g_0 = 1$  the collective mode frequencies as given by Bogoliubov-de Gennes linearization of the Gross-Pitaevskii equation (14) around our initial ground state are slightly shifted from the non-interacting frequencies; they are shown as dotted vertical lines in Fig. 6. In the Thomas-Fermi limit where there is an infinitely large interaction strength, we anticipate that the heating spikes would appear close to the eigenfrequencies  $\Omega_n^{\text{TF}} = \sqrt{n(n+1)}/2\omega$ . It is clear that most of our heating spikes are appearing very close to these collective mode resonances. The reflection symmetry of our trapping potential and initial state means that only even-parity collective modes can be excited by our driving.

Heating spikes at odd-parity Bogoliubov-de Gennes frequencies, or in between frequencies, are due to subharmonic excitation with  $m > 1$ . With  $\gamma = 1.5$  our periodic driving is significantly but not extremely anharmonic, so that subharmonics with large  $m$  or of higher modes do not seem to cause significant heating, but subharmonics with  $m = 2$  or  $m = 3$  are clearly visible for some of the lower modes, as seen in Fig. 6(b). The moderate driving amplitude of

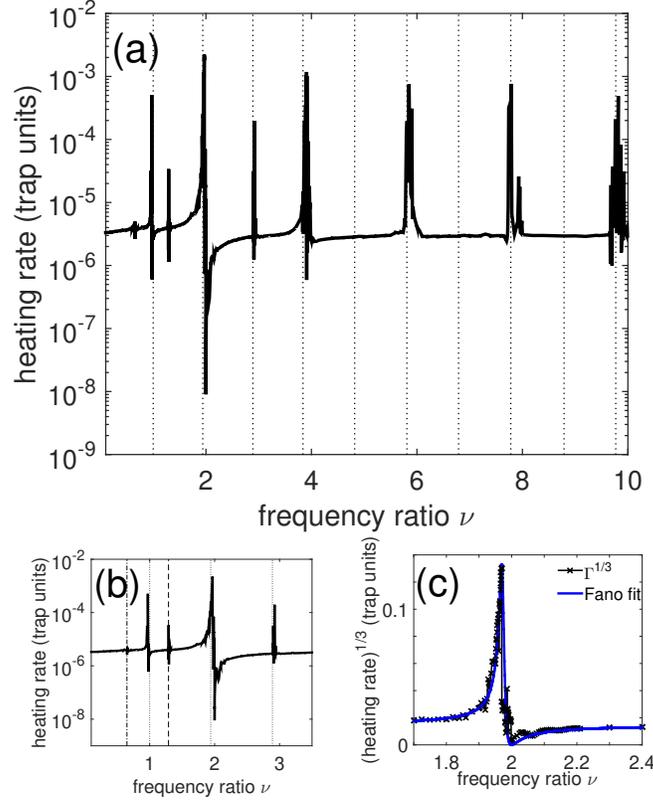


Figure 6: Heating rate as a function of driving frequency  $\nu\omega$  for runtimes taken in the long-time regime; we used the interaction strength  $g_0 = 1.0$  and driving strength parameter  $\gamma = 1.5$ . The horizontal axis does not really begin at  $\nu = 0$ , but at  $\nu = 0.05$  (the lowest value we simulated), and so the quasi-static limit of very small  $\nu$  does not appear in the plot. Dotted lines correspond to Bogoliubov – de Gennes excitation frequencies while dash-dotted and dashed lines correspond to the  $1/3$  and  $1/2$  of these frequencies, respectively. (a) Heating rate in the frequency spectrum in the semilog plot; (b) A zoom of the range of smaller frequencies where subharmonics are excited; (c) The heating rate to the power  $1/3$  is fitted to a Fano resonance peak around the exact heating zero at  $\nu = 2$ .

368  $\gamma = 1.5$  is also evidently sufficient to produce slight nonlinear shifting of the resonance peaks  
 369 away from the linear Bogoliubov-de Gennes resonances. We realize that the heating spikes  
 370 lower and broaden as the frequency is getting higher. In addition the heating rate slowly  
 371 softens on average as one would naturally expect when the frequency increases.

372 **Heating trough.** As noted above, there must be a trivial heating minimum around  $\omega = 0$   
 373 , because for very slow modulation of  $g$  the system will react adiabatically; this trough is not  
 374 seen in Fig. 6 because we do not actually show results below  $\nu = 0.05$ . Our focus is not on  
 375 the adiabatic limit, but rather on the non-trivial heating minima such as the one at  $\nu = 2$ ,  
 376 which is the zero-heating case that was identified above by our mapping. From the zoomed-in  
 377 Fig. 6c) it is clear that this zero-heating case is not a unique point, but rather the bottom of  
 378 a finite heating trough of low heating rates.

379 **Fano resonances.** Very close to this heating trough, furthermore, there is a huge heating  
 380 spike which may suggest the existence of a hidden compensation mechanism leading to the  
 381 heating suppression. In the low heating region, the system evidently responds to the drive  
 382 in a nearly adiabatic way. Even though adiabatic following is normally observed only for  
 383 slow external driving, the heating trough thus reveals a kind of hidden adiabaticity in a  
 384 rapidly driven many-body system. The appearance of the heating trough close to a heating  
 385 spike, forming a distinctly asymmetric trough-peak pattern, reminds us of the so-called *Fano*  
 386 *resonance* that occurs in nanoscale structures [46, 47]. Even though very mild oscillations of  
 387 the actual heating curve  $\Gamma(\nu)$  can be seen during the sharp resonance and heating trough, the  
 388 curve is fit quite well by the Fano function, which in this case is given by

$$\Gamma^{1/3} = 1.55 \sigma \frac{(\nu - 2)^2}{\sigma^2 + (\nu - 2 - \delta)^2}, \quad (18)$$

389 where the dimensionless width is  $\sigma = 1/105$  and the asymmetry is due to  $\delta = -0.027$ . This  
 390 Fano profile is shown in Fig. 6(c). Fano resonances are the result of an interference between  
 391 an excitation of a single mode and an excitation of a broad spectrum of modes [47]. Such a  
 392 process occurs for example in atomic physics, in the excitation of an electronic configuration  
 393 that has an energy higher than that needed to ionize the atom. Fano resonances generally  
 394 appear in the context of single-particle systems where Floquet theory is applicable to the linear  
 395 equations that describe the system [50, 51]. In those systems, Fano resonances at dynamically  
 396 created bound states in the continuum may lead to points of zero transmission where the  
 397 so-called quantum resonance catastrophe occurs. In this work, however, similar resonances  
 398 happen in the realm of many-body physics where interactions between the atoms are normally  
 399 expected to yield a more complex behavior. A deeper investigation of such resonances would  
 400 require elaborate methods that are beyond the scope of this paper but may be addressed in  
 401 future work; here we simply observe that our numerical mean-field results seem to suggest  
 402 their existence in periodically driven interacting Bose gases.

## 403 5 Mapping interaction modulation to trap modulation

404 The ‘heating rate spectroscopy’ of section 4 has identified Fano-like resonances in the heating  
 405 rate as a function of the frequency with which the contact interaction strength of a dilute  
 406 Bose gas is modulated, while the gas remains trapped in static harmonic potential of trap  
 407 frequency  $\omega$ . The resonances appear near collective mode frequencies and their subharmonics;  
 408 close beside these resonance peaks are narrow minima (troughs) in the heating rate. At the  
 409 special heating trough at drive frequency  $\omega_{\text{drive}}/\omega = \nu = 2$ , the heating rate falls all the way to  
 410 zero, as the exact mapping described in section 3 demands. While it is only this single special  
 411 heating trough that is mapped exactly onto a static experiment, we can still ask whether  
 412 the mapping may shed further light on heating in many-body Floquet systems, by mapping  
 413 experiments with  $\nu \neq 2$  onto other experiments which might be easier to analyze.

### 414 5.1 Mapping to experiments with constant interaction strength

415 We therefore now consider the evolutions of section 4 as A experiments, with constant trap  
 416 frequency  $\omega_A = \omega$  and modulated interaction strength  $g_A(t) = g(t)$  as given by (15), with

417 arbitrary overall interaction strength  $g_0$ , modulation amplitude  $\gamma$ , and drive frequency  $\nu\omega$ .  
 418 The exact mapping (2) with

$$\lambda(t) = \sqrt{\frac{1-\gamma^2}{2} \cos(\nu\omega\tau(t)) + \frac{1+\gamma^2}{2}} \implies \tan\left(\frac{\gamma\nu\omega}{2}t\right) = \gamma \tan\left(\frac{\nu\omega}{2}\tau\right) \quad (19)$$

419 then yields a B experiment for every value of  $\nu$  in which the interaction strength has been  
 420 mapped to the *time-independent*  $g_B = g_A(\tau)/\lambda(t) \equiv g_0$  but, except for  $\nu = 2$ , the trap  
 421 frequency  $\omega_B(t)$  is now a periodic function:

$$\omega_B^2(t) = \omega^2\lambda^4 + \lambda^3 \frac{d^2\lambda}{d\tau^2} = \omega^2 \left( \frac{1 - \frac{\nu^2}{4}}{\left(\frac{1+\gamma^{-2}}{2} + \frac{1-\gamma^{-2}}{2} \cos(\gamma\nu\omega t)\right)^2} + \frac{\nu^2\gamma^2}{4} \right). \quad (20)$$

422 In the case  $\nu = 2$  we thus recover the entirely static experiment with trap frequency  $\gamma\omega$ ,  
 423 from which we constructed the heating-free driven experiment in the first place, via the inverse  
 424 of this mapping. For  $\nu \neq 2$  we now have a class of experiments with constant interactions and  
 425 modulated trapping frequencies, which are mapped exactly as quantum many-body problems  
 426 onto the experiments that we analyzed in mean-field theory in section 4. We still cannot solve  
 427 these new evolution problems exactly, and solving them in mean-field theory will only yield the  
 428 image under the mapping of our results from section 4 above. It might be possible, however,  
 429 to obtain some insight or intuition about our heating peaks and troughs by considering these  
 430 physically quite different experiments which are exactly mapped versions of the previous ones.

## 431 5.2 Mapping the mean-field energy

432 We can establish that a B experiment will show long-term heating if and only if the cor-  
 433 responding A experiment shows long-term heating. This is intuitive but not quite obvious,  
 434 because it is straightforward to show that the mapping (2) transforms the Gross-Pitaevskii  
 435 mean-field energy (16) as

$$\begin{aligned} E_B(t_B) &= \lambda^2 E_A(t_A) + \frac{M}{4} \left( \frac{d^2\lambda^2}{dt_A^2} \right) \int dx x^2 |\psi_A(x, t_A)|^2 \\ &\quad - \frac{M}{4} \left( \frac{d\lambda^2}{dt_A} \right) \frac{d}{dt_A} \int dx x^2 |\psi_A(x, t_A)|^2 \end{aligned} \quad (21)$$

436 for  $t_A = \tau(t_B)$ . Thus the energies in the two experiments are not simply the same.

437 Suppose, however, that  $E_A$  does not show secular growth. In this case  $\int dx x^2 |\psi_A|^2$  cannot  
 438 show secular growth, either, because  $E_A$  is a sum of positive definite terms, one of which  
 439 is proportional to  $\int dx x^2 |\psi_A|^2$ , so that if  $\int dx x^2 |\psi_A|^2$  grew secularly then  $E_A$  would have  
 440 to grow secularly as well. Thus if  $E_A$  does not show secular growth, then neither can  $E_B$ ,  
 441 because it consists only of non-growing terms multiplied by periodic functions.

442 Suppose now that  $E_B$  does not show secular growth. As we observed above, it is straight-  
 443 forward to confirm from our mapping definition (2) that the inverse transformation which  
 444 maps from B to A is simply the mapping with  $\lambda \rightarrow 1/\lambda$ . Hence we also have

$$\begin{aligned} E_A(t_A) &= \lambda^{-2} E_B(t_B) + \frac{M}{4} \left( \frac{d^2\lambda^{-2}}{dt_B^2} \right) \int dx x^2 |\psi_B(x, t_B)|^2 \\ &\quad - \frac{M}{4} \left( \frac{d\lambda^{-2}}{dt_B} \right) \frac{d}{dt_B} \int dx x^2 |\psi_B(x, t_B)|^2. \end{aligned} \quad (22)$$

445 Hence by the same argument that we have just made above, if  $E_B$  does not show secular  
 446 growth then neither can  $E_A$ . Thus it is impossible for  $E_A$  to grow secularly without  $E_B$  also  
 447 growing secularly.

448 The questions of secular heating in A and B experiments are therefore really both the  
 449 same question. Whatever mechanisms cause or suppress heating in one kind of experiment  
 450 will be the images, under our mapping, of the mechanisms that cause or suppress heating in  
 451 the other kind of experiment. Unfortunately, however, this does not necessarily mean that  
 452 the mechanisms are obvious in either case. To illustrate the kind of subtle problem that can  
 453 occur even in the comparatively simple B experiments, in which only the trap potential is  
 454 modulated, we will propose an argument for explosive heating in some A experiments, which  
 455 should seem plausible but has in fact already been disproven by our results in section 4 above.

### 456 5.3 Absence of Bose fireworks from intermittent anti-trapping

457 **Repulsive potentials.** For  $\gamma \neq 1$  and  $\nu$  above a  $\gamma$ -dependent threshold  $\nu_-(\gamma)$ ,  $\omega_B^2(t)$  as  
 458 given by (20) can become negative within certain time intervals. The threshold  $\nu$  above which  
 459  $\omega_B^2 < 0$  is

$$\nu > \nu_-(\gamma) = \begin{cases} \frac{1}{\sqrt{1-\gamma^2}} & , \quad \gamma < 1 \\ \frac{\gamma}{\sqrt{\gamma^2-1}} & , \quad \gamma > 1. \end{cases} \quad (23)$$

460 There are no values of  $\gamma$  and  $\nu$  for which the trapping strength  $\omega_B^2$  becomes negative for all  $t$ ,  
 461 but for  $\nu$  above a higher threshold  $\nu_c(\gamma)$  the *average* trap strength over a driving period does  
 462 become negative:

$$\begin{aligned} \langle \omega_B^2 \rangle &= \frac{\gamma\omega\nu}{\pi} \oint \omega_B^2(\tau) d\tau = \frac{\gamma\omega^2}{2} \left( 1 + \gamma^2 - \frac{\nu^2}{4}(1-\gamma)^2 \right) \\ \implies \nu_c(\gamma) &= 2 \frac{\sqrt{1+\gamma^2}}{|1-\gamma|} \equiv \nu_c(1/\gamma). \end{aligned} \quad (24)$$

463 **Bose fireworks?** It seems plausible that heating should increase in some significant way  
 464 when  $\nu > \nu_-(\gamma)$ , since then the quasi-one-dimensional gas is being repeatedly subjected to  
 465 a repulsive potential instead of a trap. And it seems plausible that heating should become  
 466 quite strong indeed for  $\nu > \nu_c(\gamma)$ , since then the gas is actually being anti-trapped, rather  
 467 than trapped, for most of the time. Moreover the thresholds  $\nu > \nu_-$  for  $\omega_B^2(t) < 0$  and  $\nu > \nu_c$   
 468 for  $\langle \omega_B^2 \rangle < 0$  have nothing to do with the mean-field approximation; they are facts about the  
 469 time-dependent potential strength  $\omega_B^2(t)$  which remain true in the full quantum many-body  
 470 problem. It may therefore not seem too much to expect that our mapped B experiments are  
 471 here predicting something like the so-called ‘Bose fireworks’ that have been seen in A-like  
 472 experiments with modulated interaction strength [48, 49].

473 **No.** In fact, however, Fig. 6 has already shown that there is no substantial increase in  
 474 the heating rate for either  $\nu > \nu_-$  or  $\nu > \nu_c$ . Whether or not this is counter-intuitive, the  
 475 particular form of modulated potential (20) simply does not cause any fireworks-like heating.  
 476 The slow, linear heating that we have seen in section 4 above does occur, as long as  $g_0 \neq 0$ ,  
 477  $\gamma \neq 1$ , and  $\nu \neq 2$ . In the B experiments this heating is produced by the modulated trapping  
 478 potential alone, with the interaction strength  $g_B = g_0$  constant. The fact that this form

479 of potential modulation does not generate abundant heat by itself, however, can be seen by  
 480 considering the non-interacting case  $g_0 = 0$ . One might expect the modulating potential to  
 481 heat the non-interacting gas, but for  $g_0 = 0$  the A experiment to which all the B experiments  
 482 can be mapped has time-independent trap strength  $\omega^2$  and also  $g_A = 0$ . There is therefore  
 483 no heating at all for  $g_0 = 0$  in either A or B experiments, for any values of  $\nu$  or  $\gamma$ .

484 We must therefore recognize that neither  $\omega_B^2(t) < 0$  nor even  $\langle \omega_B^2 \rangle < 0$  has to imply  
 485 dramatic heating. Evidently in the intervals of positive  $\omega_B^2$  the trap can be strong enough to  
 486 pull the gas back together again after it has been dispersed during the anti-trapping intervals  
 487 of negative  $\omega_B^2$ .

488 The analogy between our A experiments with modulated interaction and the actual ‘Bose  
 489 fireworks’ experiments is also evidently not as close as it might at first seem. First of all  
 490 in [48, 49] the interaction strength  $g(t)$  was much more strongly modulated than our  $g(t)$   
 491 from (15) can allow for any  $\gamma$ : the real experiments had  $g(t)$  oscillating between positive and  
 492 negative values, with an amplitude some twelve times greater than the mean value of  $g(t)$ .  
 493 With large  $\gamma$  we can achieve arbitrarily large amplitude in  $g(t)$  but our (15) does not allow  $g(t)$   
 494 to change sign. Secondly the real experiments used a trapping potential of finite depth, and  
 495 for highly excited atoms this is qualitatively different from our parabolic potential extending  
 496 to infinity.

## 497 6 Conclusion and outlook

498 Periodically driven many-body quantum systems are a useful experimental tool for under-  
 499 standing non-equilibrium physics, but one phenomenon of non-equilibrium physics which they  
 500 cannot in general avoid is the problematic phenomenon of secular heating. Our exact space-  
 501 time mapping between quantum fields provides a limited range of results for such problems,  
 502 but in the special cases that the mapping provides, it is exact, and so it can supply us with  
 503 instructive examples. We have used it to identify a special form of periodic modulation of the  
 504 strength of the contact interaction in a dilute Bose gas: a special case in which there is no  
 505 heating at all. This shows the possibility of a kind of hidden adiabaticity in rapidly driven  
 506 interacting quantum systems.

507 We have further explored this phenomenon with numerical calculations for quasi-one-  
 508 dimensional Bose-Einstein condensates in Gross-Pitaevskii mean-field theory, showing that  
 509 the heating rate in this kind of system can show Fano-like resonances as a function of driving  
 510 frequency. The exact zero-heating case found by our mapping appears to be one of these  
 511 resonances, but many similarly narrow, deep troughs in the heating rate can also appear.  
 512 From our failed speculations about dramatic heating in section 5, the cautionary lesson must  
 513 be drawn that the exact mapping may not always be able to simplify complex experiments by  
 514 mapping them onto simple ones. Sometimes it will instead reveal that the seemingly simple  
 515 experiments are not really as simple as they seemed. This is also learning something, however.  
 516 Further applications of the exact spacetime mapping of quantum fields [32, 33] to periodically  
 517 driven quantum many-body systems will be well worth pursuing.

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