Superradiant Solid in Cavity QED Coupled to a Lattice of Rydberg Gas

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SUPPLEMENTARY MATERIAL

A. Two-photon transition

To describe the two-photon process, we begin with the following Hamiltonian (set $E_{\mathrm{g}} = 0$) for a single site i in the rotating-wave approximation:

$$\begin{aligned} H_i &= E_p|p_i\rangle\langle p_i| + E_e|e_i\rangle\langle e_i| + \{\hbar g_0\hat{\psi}^\dagger|\mathrm{g}_i\rangle\langle p_i| + H.c.\} \\ &+ \hbar\omega_c\hat{\psi}^\dagger\hat{\psi} + \{\hbar\Omega\exp(-i\omega_l t)|e_i\rangle\langle p_j| + H.c.\}. \end{aligned} \tag{1}$$

We first perform a unitary state transformation $\hat{U}(t) = e^{i\hat{S}t}$ with $\hat{S} = \exp\{\hbar\omega_c(\hat{\psi}^\dagger\hat{\psi} + |p_i\rangle\langle p_i|) + \hbar(\omega_c + \omega_l)|e_i\rangle\langle e_i|\}$, the above Hamiltonian transforms as $\tilde{H} = \hat{U}H\hat{U}^\dagger - \hat{S}$, which results in

$$\begin{aligned} \tilde{H}_i &= -\hbar\Delta_p|p_i\rangle\langle p_i| - \hbar\Delta|e_i\rangle\langle e_i| + \{\hbar\Omega|e_i\rangle\langle p_i| + H.c.\} \\ &+ \{\hbar g_0\hat{\psi}^\dagger|\mathrm{g}_i\rangle\langle p_i| + H.c.\}. \end{aligned}$$

Here $\Delta_p = \omega_c - E_p$ is the detuning between the cavity field and $|\mathrm{g}\rangle \to |p\rangle$ transition, $\Delta = \omega_c + \omega_l - E_e \equiv \Delta_p + \Delta_e$ is the total detuning with $\Delta_e = \omega_l - (E_e - E_p)$ denoting the detuning between the Rabi laser and $|p\rangle \to |e\rangle$ transition.

Assuming a large detuning from the intermediate state, which satisfies $\Delta_p \gg \Gamma$ with Γ^{-1} the lifetime of the itemediate $|p\rangle$ state, we can adiabatically eliminate the unpopulated intermediate state and get the effective Hamiltonian

$$H_i^{\mathrm{eff}} = -\hbar\Delta|e_i\rangle\langle e_i| + \{\hbar\frac{g_0^*\Omega}{|\Delta_p|}\hat{\psi}|e_i\rangle\langle \mathrm{g}_i| + H.c.\}. \tag{2}$$

Then, we go back to a new rotating frame by the unitary transformation $\exp\{-it\omega_c(\hat{\psi}^\dagger\hat{\psi} + |e_j\rangle\langle e_j|)\}$, we arrive at the following effective two-level atom-cavity coupled Hamiltonian

$$\hat{H}_i = \omega_c\psi^\dagger\psi + \epsilon|e_i\rangle\langle e_i| + g(\psi|e_i\rangle\langle g_i| + \mathrm{H.c.}), \tag{3}$$

with $g = \hbar g_0^*\Omega_l/|\Delta_p|$ the effective coupling constant and $\epsilon = \omega_c - \Delta$ the effective transition frequency.

Finally, we extend the above two-photon transition process to a 1D lattice of atoms coupled to the cavity field, we derive the generalized Dicke model Hamiltonian (1) in the context. Here, we should mention that each atom of the 1D lattice can be excited independently which is in contrast to Ref. [24], where all the N atoms are in the blockade regime and excited collectively with only one Rydberg state.

B. Quantum Monte Carlo simulation

Here we describe briefly on Stochastic Cluster Series Expansion (SCSE) method [30], which we extended to our atom-light coupled system. The main steps are as follows. First, we split the Hamiltonian with the three-sites operators $h_{i,\nu}$. Then, by the series expansion, the partition function reads

$$\begin{aligned} Z &= \sum_{M,\alpha,\{\hat{\mathbb{O}}\}} \frac{(-\beta)^M}{M!}\langle\alpha|\prod_{m=1}^{M}\hat{O}_m|\alpha\rangle \\ &= \sum_{M,\{\hat{\mathbb{O}}\},\{\vec{\alpha}\}} \frac{(-\beta)^M}{M!}\prod_{m=1}^{M}\langle\alpha_m|\hat{O}_m|\alpha_{m+1}\rangle. \end{aligned}$$

Here, $\{\vec{\alpha}\} = \{\alpha_1, \alpha_2, \ldots, \alpha_M\}$ are all possible intermediate states with $\alpha_1 = \alpha_{M+1}$. $\{\hat{\mathbb{O}}\} = \{\hat{O}_1, \hat{O}_2, \ldots, \hat{O}_M\}$ represent all possible sequences of $\hat{O}_m$, with $\hat{O}_m$ being any three-sites operator $\{h_{i,\nu}\}$. The off-diagonal three-sites operator $h_{i,\nu}$

are given by

$$\begin{aligned} h_{i,1} &= \frac{\bar{g}}{2\sqrt{N}}\psi b_i^\dagger a_i, \\ h_{i,2} &= \frac{\bar{g}}{2\sqrt{N}}\psi^\dagger b_i a_i^\dagger, \\ h_{i,3} &= \frac{\bar{g}}{2\sqrt{N}}\psi b_{i+1}^\dagger a_{i+1}, \\ h_{i,4} &= \frac{\bar{g}}{2\sqrt{N}}\psi^\dagger b_{i+1} a_{i+1}^\dagger, \end{aligned}$$

and the diagonal operator

$$h_{i,5} = -\frac{\mu-\omega_c}{N}\psi^\dagger\psi - \frac{\mu-\epsilon}{2}(P^{(i)}+P^{(i+1)}) + VP^{(i)}P^{(i+1)}.$$

To calculate the weights of the above partition function Z, we take $h_{i,1}$ as an example.

	$\lvert\alpha_m\rangle$	operator	$\lvert\alpha_{m+1}\rangle$
atom excitation on site i	1		0
photon	$n-1$	$h_{i,1}$	n
atom excitation on site $i+1$	1		1

As seen in the Table above, the weight $\langle\alpha_m|h_{i,1}|\alpha_{m+1}\rangle$ can be represented as a diagram $\frac{1}{0}\ \frac{n-1}{n}\ \frac{1}{1}$ in imaginary time m, and all the corresponding weights in the partition function Z form a diagram representation. Here the single-occupation constraint of the atomic excitation has been taken into account.

Then, we perform the conventional directed-loop SSE algorithm to alter the diagram, and employ the Metropolis to sample the partition function. Finally, the mean value of various observables can be calculated from these samples. We should mention that the photon field may introduce truncation error to the system. To deal with this point properly, we set the initial maximum photon number to be $n_{\max}$. Then during the QMC simulation, once $n_{\max}$ is reached, it will be raised and the simulation will be restarted at the same time.

C. The finite size scaling

In this section, we perform the finite size scaling for the generalized DM. Although the real experimrnt will be implemented for finite size with about 100 lattice sites in a cavity, one may wonder whether the new phases predicted in the our work exist in the infinite system. For this purpose, we have also made a finite size scaling analysis for the SRS phase. Figure 1 shows the corresponding scaling results in the SRS phase with lattice size $N = 100$, 150, 200, 250 and 300. we can find that both $S(Q)/N$ and κ_T scale proportional to N^{-1} and converge to finite values in the infinite size limit.

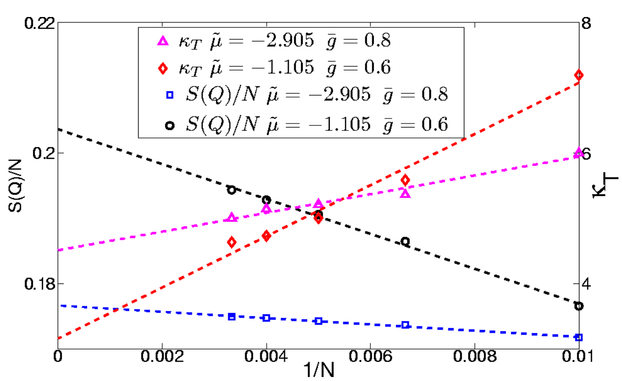

FIG. 1: (color online). The finite size scaling for $S(Q)/N$ and κ_T at $V = 1$, $\Delta = 3$ and $\beta = 500$ vs different sizes. $\tilde{\mu} = -2.905$ (-1.105) correspond to the SRS phases on the trajectories of the lower (upper) cuts shown in Fig.3 (a).

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