Eggert, Affleck, and Horton Reply: In the preceding Comment [1] it is pointed out correctly that the field theory treatment that was used in our recent Letter [2] to obtain some of the results for the Heisenberg antiferromagnetic chain is indeed valid only in the limit of long length L, low temperature T, and small magnetization S^z. In particular, this treatment becomes only asymptotically correct in a region where the dispersion is linear and the spin-wave velocity v can be approximated by a constant [3], which according to our numerics is the case if both $T \lesssim 0.2J$ and $L \gtrsim 10$ sites. There is no restriction on the product LT/v as long as v is approximately constant.

However, we must emphasize that we were indeed able to calculate the staggered susceptibility χ_1 for *arbitrary* L and T as mentioned in the introduction by combining the field theory results with numerical calculations [4]. The numerical calculations are especially reliable for values of L and T where the field theory predictions become invalid and vice versa. We can therefore describe the entire crossover of χ_1 to the limit of large T and/or small L, which shows an interesting behavior by itself that was unfortunately not explicitly presented in the Letter [2]. If we, for example, consider the staggered susceptibility χ_1 without impurities as a function of T, we see that it crosses over from the bosonization formula to a high temperature expansion as shown in Fig. 1.

$$\chi_1(T) \to \begin{cases} \frac{b\sqrt{\ln(a/T)}}{T} & T \ll J \\ \frac{1+J/2T}{4T} & T \gg J \end{cases}, \tag{1}$$

where $a \sim 23J$ and $b = \frac{\Gamma^2(1/4)}{4\sqrt{2\pi^3}\Gamma^2(3/4)} \approx 0.277\,904$. In the case of shorter chain lengths L, we again find a significant drop from the thermodynamic limit as well as a split at $T \lesssim 4J/L$ for even and odd chains as depicted for $L = 10$ and $L = 11$ in Fig. 1. The crossover from finite size behavior to the thermodynamic limit is therefore very similar to Fig. 1 in our Letter [2], which shows the behavior predicted by bosonization in the limit $L \to \infty$, $T \to 0$ as a function of LT, compared to numerical results for large L. Even for smaller L, we find again that $\chi_1(T, L) \propto L$ for even chains as $T \to 0$ and $\chi_1(T, L) \to c/T$ for odd chains, where the intercept c can be approximated by a length independent constant even down to $L = 1$ as shown in the inset in Fig. 1.

Now that we have displayed χ_1 for arbitrary T, we may be tempted to again apply the chain mean field equation

$$zJ'\chi_1(T_N) = 1 \tag{2}$$

even in the case where J' is of the order of J. Although we might not expect any one-dimensional physics to survive in that limit, we find, for example, that this would result in $T_N \approx 1.386J$ for a simple cubic lattice with $J = J'$, which is indeed higher than the accepted values [5], but still an improvement over the ordinary mean field result of $T_N = 1.5J$. If J' is of order J, only extreme doping levels will significantly affect the ordering temperature, since finite size effects are small at higher temperatures $T \gtrsim 4J/L$. In conclusion, we have calculated the staggered susceptibility for arbitrary L and T and outlined in more detail the behavior in the limit of large T and small L.

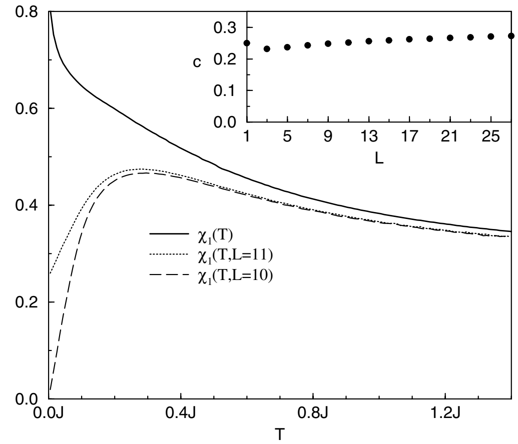

FIG. 1. The staggered susceptibility $\chi_1(T)$ in the thermodynamic limit determined by combining bosonization results at lower temperature and numerical simulations at higher temperature. The numerical results for $L = 10$ and $L = 11$ are also shown. Inset: the intercept $c = \lim_{T\to 0} T\chi_1(T, L)$ as a function of L.

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