Supplementary Material for "Continuous easy-plane deconfined phase transition on the kagome lattice"

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In this supplementary material, we analyze the finite size behavior of anomalous exponents, which are extracted from two point correlators. The resulting size dependent exponents also describe the behavior of the condensate fraction and the structure factor, while fixed exponents extracted from a data collapse are not reliable and show strong deviations. Then, we show data collapse of superfluid density with two-length scales scenario. At last, We also implement the numerical flowgram method to study the phase transition.

I. FINITE SIZE SCALING

At the critical point of a second order phase transition, the two-point correlator follows a power lay decay $C_L(r) = a(L)r^{-1-\eta(L)}$. In large systems, the structure factor or condensate fraction is proportional to its integration per site:

$$\int_1^L C_L(r)rdr/L^2 = \frac{a(L)}{1-\eta(L)}(L^{-1-\eta(L)} - L^{-2}). \quad (1)$$

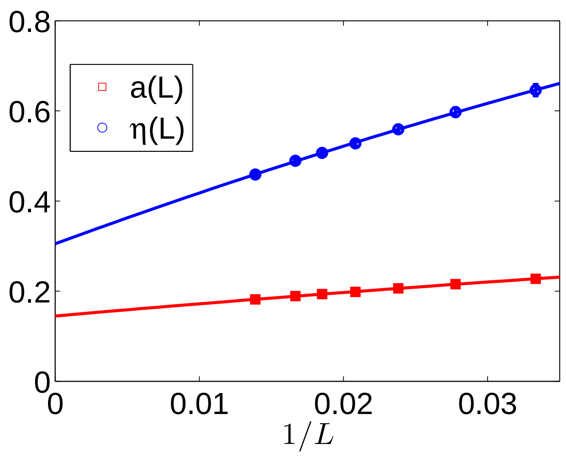

FIG. 1. The coefficients $a(L)$ and $\eta(L)$ got from off-diagonal two-point correlator in different sizes, by using power fitting $C_L(r)$ at 1/3 filling, $t/V = 0.1303$ and $\beta V/L = 25/3$.

As mentioned in main text, we can use $C_L(r)$ to fit the two-point correlator function. For the off-diagonal correlator $b_i^\dagger b_j$ in Fig.1, we can find the prefactor $a(L)$ changes less than $\eta(L)$. Then, we use a second order polynomial function $\eta_f(L)$ to fit $\eta(L)$:

$$\eta_f(L) = \eta_0 + \eta_1/L + \eta_2/L^2, \quad (2)$$

which gives $\eta_0 = 0.305(0.020)$. We also considered higher order polynomial fitting, but the third order $\eta_0 = 0.299(0.113)$ has much less accuracy and higher orders are even overfitted.

In order to check how finite size effects change the anomalous exponent, we substitute $\eta(L)$ with fitting function η_f in Eqn.(1) and get

$$f_s \equiv \frac{a_f(L)}{1-\eta_f(L)}(L^{-1-\eta_f(L)} - L^{-2}), \quad (3)$$

where $a_f(L)$ is a second order polynomial fitting function for $a(L)$, and condensate fraction should be proportional to f_s. Then, we analyze the finite size effect of $a_f(L)$, L^{-2} and $\eta_f(L)$ separately by defining:

$$f_a \equiv \frac{a_0}{1-\eta_f(L)}(L^{-1-\eta_f(L)} - L^{-2}), \quad (4)$$

$$f_2 \equiv \frac{a_0}{1-\eta_f(L)}(L^{-1-\eta_f(L)}), \quad (5)$$

$$f_\infty \equiv \frac{a_0}{1-\eta_0}(L^{-1-\eta_0}). \quad (6)$$

As shown in Fig.2, f_s matches well with ρ_0 after rescaling its magnitude ($\rho_0^{RN} = C\rho_0$). Both a_f and L^{-2} terms can change the shape of the curve. However, the finite size effect of η_f bends the curve which explains why the anomalous critical exponent obtained from a data collapse of the condensate fraction deviates strongly. A similar phenomenon also happens for the density correlator and structure factor show in Fig.3 and Fig.4. Therefore the size independent critical exponents got directly from structure factor and condensate fraction are less convincing.

In addition, we also consider the data collapse of superfluid density with two-length scale scenario. As shown in Fig.5, it is reasonable as good as LC scenario, so we can not conclude which one is better.

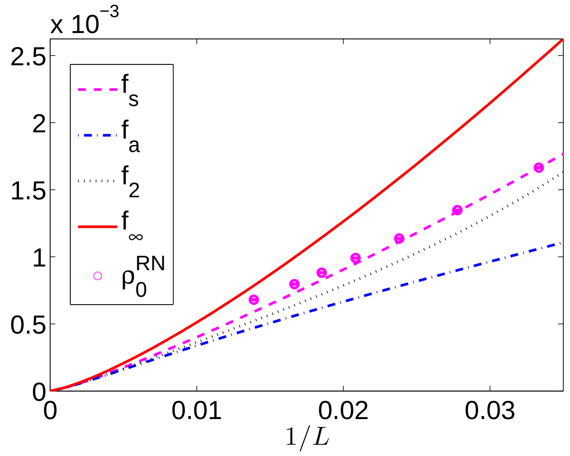

FIG. 2. Comparing condensate fraction with different functions which neglects various size effect terms

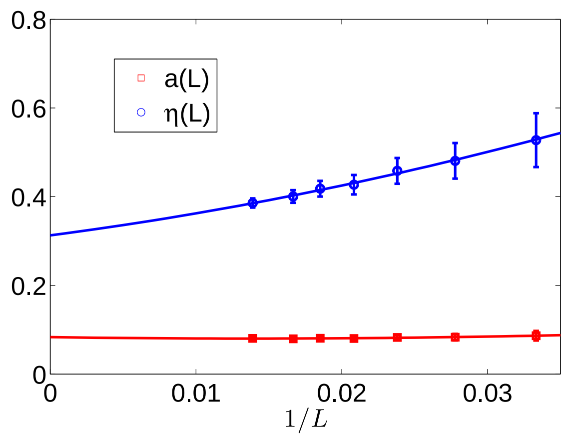

FIG. 3. The coefficients $a(L)$ and $\eta(L)$ got from density two-point correlator in different sizes, by using power fitting $C_L(r)$ at 1/3 filling, $t/V = 0.1303$ and $\beta V/L = 25/3$.

II. NUMERICAL FLOWGRAM

The numerical flowgram method was introduced by A.B. Kuklov, *et. al.* [1–3] to study the DCPs. They obtain the finite size critical point $t_c(L)/V$ from the condition that the ratio of probabilities of having zero and non-zero winding numbers is some fixed number of the order of unity. If the transition is continuous, they claim the winding number at this critical point will approach a universal value when enlarging the system, otherwise, it will linearly scale with system size L.

We also implement numerical flowgram method for our case. We fix the probability of zero winding numbers P_0 equal to 0.25 which marked in Fig. 6. We determine the size-dependent hopping values $t_{P_0}(L)$ for this probability. The corresponding winding numbers for those parameters can then be analyzed as a function of length L. As shown in inset of Fig.6, the winding numbers don't

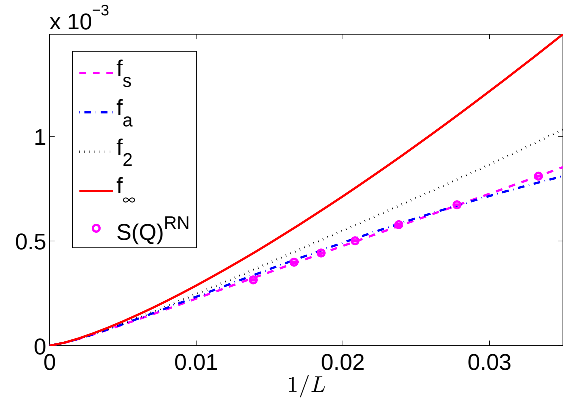

FIG. 4. Comparing structure factor with different functions which neglects various size effect terms.

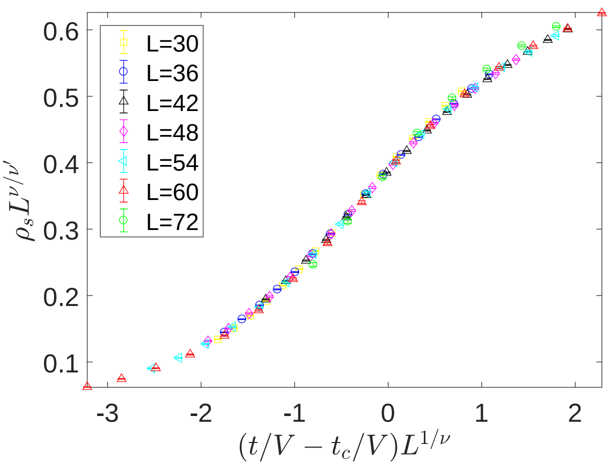

FIG. 5. Data collapse of superfluid density with two-length scales scenario at 1/3 filling and $\beta V/L = 25/3$. The critical point is $t_c/V = 0.130263(0.000003)$, and the critical exponents are $1/\nu = 2.299(0.041)$ and $\nu/\nu' = 0.524(0.014)$.

flow to a universal value, but seem to linearly depend on the system size. Similar behaviour is also found in the J-Q model [3], and it may be directly related to the drift of the superfluid density or weakly first order phase transition.

[1] A. Kuklov, N. V. Prokof'ev, B. Svistunov, and M. Troyer, Annals of Physics **321**, 1602 (2006).
[2] A. B. Kuklov, M. Matsumoto, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Phys. Rev. Lett. **101**, 050405 (2008).
[3] K. Chen, Y. Huang, Y. Deng, A. B. Kuklov,

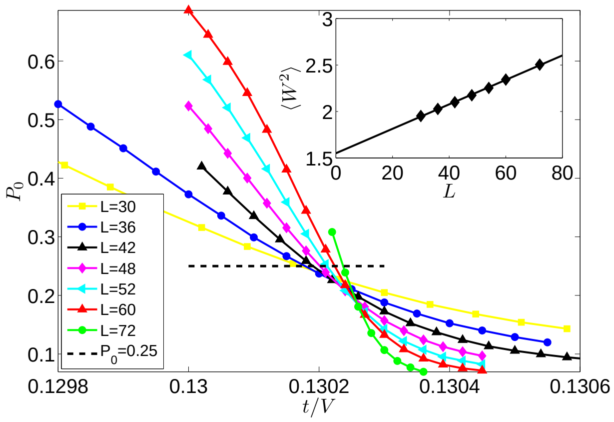

FIG. 6. The possibility of zero winding number vs t/V for different sizes at 1/3 filling and $\beta V/L = 25/3$. Inset: The winding numbers for different size at finite size critical points where $P_0 = 0.25$ (dash line in main panel).

N. V. Prokof'ev, and B. V. Svistunov, Phys. Rev. Lett. **110**, 185701 (2013).