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Interference and interactions in mesoscopic rings

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Abstract

We consider a mesoscopic ring connected to external reservoirs by tunnel junctions. The ring is capacitively coupled to an external gate electrode and may be pierced by a magnetic field. Due to strong electron–electron interactions within the ring the conductance shows Coulomb blockade oscillations as a function of the gate voltage, while Aharonov–Bohm interference effects lead to a dependence on the magnetic flux. The Hamiltonian of the ring is given by a Luttinger model that allows for an exact treatment of both interaction and interference effects. We conclude that the positions of conductance maxima as a function the external parameters can be used to determine the interaction parameter g, and the shapes of conductance peaks are strongly affected by electron correlations within the ring.

Keywords: Luttinger liquid; Coulomb blockade; Aharonov–Bohm oscillations

Mesoscopic devices have proven to be a rich source of novel physical phenomena. Originally, the main focus were numerous *interference* effects like weak localization, universal conductance fluctuations, and persistent currents. Later it was realized that in the same mesoscopic systems *interactions* give rise to equally interesting physics like Coulomb blockade and strong correlations. In this article, we consider a particular experimental arrangement where purely quantum-mechanical interference effects and essentially classical interaction effects combine in a way that can be utilized to study the electronic properties of mesoscopic systems in more detail.

We consider a device consisting of a small ring (circumference L) of interacting electrons connected to two non-interacting reservoirs by tunnel junctions (see inset of Fig. 1). The left and right tunnel junctions are at positions x_{L} and x_{R}, respectively. The ring is capacitively coupled to an external gate electrode and may be pierced by a magnetic flux. We consider a small AC voltage applied to the right lead and wish to evaluate the current at the left junction.

We describe the ring using the spinless Luttinger model. In the bosonized form the Hamiltonian reads [1]

$$H_{\mathrm{ring}} = \frac{\pi\hbar}{2L}\left[\frac{v}{g}(\hat{N}-N_0)^2 + gv(\hat{J}-J_0)^2\right] + \sum_{q\neq 0}\hbar v|q|b_q^{\dagger}b_q,$$

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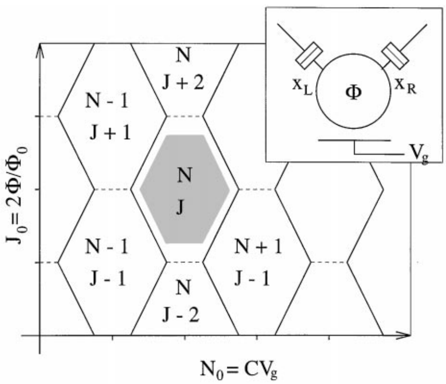

Fig. 1. Positions of conductance resonances in the (V_g, Φ)-plane for interacting electrons (repulsive interactions, $g = 1/\sqrt{2}$). The labels N and J denote the ground-state charge and current as a function of the external parameters, and the shaded area indicates the domain of validity of our analysis. The line segments with different slopes correspond to fluctuations in the numbers of clockwise and counterclockwise moving electrons, respectively. Inset: device geometry.

where $\hat{N}$ and $\hat{J}$ are zero modes associated with the total charge and total current. Since the numbers of clockwise and counterclockwise moving electrons on the ring must both be integers, the quantum numbers N and J must satisfy $(-1)^N = (-1)^J$. The gate voltage and magnetic flux determine the parameters $N_0 = CV_g/e$ and $J_0 = 2\Phi/\Phi_0$ which in turn determine the ground-state charge and current. The parameter g is a measure of the interaction strength, and equals one for non-interacting electrons [2]. For later convenience we define $\gamma = \frac{1}{2}(g + g^{-1}) - 1$ which vanishes in the non-interacting limit and is positive otherwise.

A straightforward application of the Kubo formula shows that the conductance is proportional to the retarded correlation function between the currents through the left and right tunnel junctions [3]. This is most readily evaluated in imaginary time. To lowest non-zero order in the tunneling Hamiltonian the current–current correlation function is related to a product of four Fermion operators in the ring, $A(\tau_1 - \tau_2, \tau_1 - \tau, \tau_2 - \tau') = \langle T_\tau(\psi^\dagger(\tau, x_L)\psi(\tau_1, x_L)\psi^\dagger(\tau_2, x_R)\psi(\tau', x_R))\rangle$. The exact evaluation of A is quite cumbersome (it consists of 144 different terms) although in principle straightforward. However, since the lowest-order expansion in terms of the tunneling Hamiltonian is valid only sufficiently far from resonance energies, only a small subset of the terms need be considered. This approximation is basically similar to the one used by Fazio and co-workers [4] for an interacting ring connected to superconducting leads. That allows us to evaluate A in an approximate fashion, and yields the DC conductance

$$\sigma \approx \frac{e^2}{h} \frac{|t_L t_R|^2}{\hbar^2} D_L(\varepsilon_F) D_R(\varepsilon_F) |G^{ret}(\omega = 0, x_L - x_R)|^2, \quad (1)$$

where $G^{ret}(\omega = 0, x_L - x_R)$ is the retarded electron Green's function in the ring. Here $t_{L/R}$ are the tunneling amplitudes between the ring and the left and right leads and $D_{L/R}$ are densities of state in the leads.

The imaginary-time Green's function $G(\tau, x)$ for interacting electrons is well known in the $T = 0, L = \infty$ case. By means of a conformal mapping we can even obtain the Greens functions for the cases $T > 0$, $L = \infty$ and $T = 0$, $L < \infty$. In the general case, the Greens function must be periodic in x and antiperiodic in τ, so it is not too surprising that $G(\tau, x)$ is in general given by a combination of doubly quasiperiodic Jacobi theta functions (elliptic functions) [5] which reduce to hyperbolic or trigonometric functions in the limits $T \to 0$ and $L \to \infty$. The Jacobi functions appear also in the partition function and, therefore, in expressions for several thermodynamic quantities like persistent currents [6].

The parameters that are most readily accessible in an experiment are the gate voltage and the magnetic flux. They enter only the $q = 0$ part of the Hamiltonian which we can re-write as $H_0 = \frac{1}{2}E_c(\hat{N} - N_0)^2 + [(\pi\hbar v_F)/2L](\hat{J} - J_0)^2$ where $v_F = gv$ is the Fermi velocity of a non-interacting system with the same density and $E_c = \pi\hbar v_F/g^2 L$ is the charging energy. The conductance resonances correspond to values of the gate voltage and magnetic flux at which the ground-state quantum numbers N and J change (degenerate ground state). In the (V_g, Φ)-plane the resonance positions form a network the shape of which depends on the interaction parameter g. Therefore, the interaction parameter can be experimentally measured by studying the trajectories of conductance maxima as a function of the gate voltage and magnetic flux.

The resonance line shape for small $\delta\varepsilon$, i.e. close to a resonance, is independent of the interaction parameter g, which is a consequence of a finite minimum energy of the bosonic modes. The resonant contribution dominates for

$$\delta\varepsilon \lesssim \delta\varepsilon_c = \frac{2\pi\hbar v}{L}\left|\sin\left(\frac{\pi(x_L - x_R)}{L}\right)\right|^{\gamma}$$

(up to logarithmic corrections); for $\delta\varepsilon \gg \delta\varepsilon_c$ the valley conductance levels off to a constant value proportional to $(a/|x_L - x_R|)^{2\gamma}$. For large separations $\Delta x = |x_L - x_R|$ the crossover point $\delta\varepsilon_c$ exceeds half of the resonance spacing and the crossover is not observed.

At $T = 0$ the smallest $\delta\varepsilon$ that we can consider is determined by when terms that are higher order in the tunneling Hamiltonian become significant. Therefore, we expect that at $T = 0$ the peak width is given by $\delta\varepsilon$ such that $\Gamma_L \Gamma_R (\sqrt{2}a/L)^{2\gamma}(\delta\varepsilon)^{-2} \approx 1$ where $\Gamma_L = |t_L|^2 D_L$. Hence, the width of a conductance peak is renormalized by the factor $(a/L)^{\gamma}$ by correlations in the ring. The valley conductance, on the other hand, is only renormalized by the factor $(a/\Delta x)^{2\gamma}$. The two renormalization factors have a simple interpretation: near a resonance the lifetime of a charged excitation of the ring is very long, all electrons in the ring must respond to the extra charge and the ring size L is important, whereas off-resonance the life time of the charged excitation is short, and only electron between the two contacts feel the presence of an extra charge.

The experimental possibilities for the study of nanostructures like the one we consider are developing rapidly. New techniques like conducting organic molecules and carbon compounds are emerging to complement the conventional semiconductor structures. In particular, it was recently demonstrated [7, 8] that carbon nanotubes exhibit coherent electron transport and can be used to fabricate nanoscale ring structures. We believe these devices can be used to experimentally study the system we have analyzed.

In conclusion, we have considered tunneling through a finite strongly interacting system within the framework of an exactly solvable model. We find that the positions of conductance resonances in the (V_g, Φ)-plane can be used to determine the interaction parameter g. We conclude that at $T = 0$ the heights of resonance peaks are unaffected by interactions but due to the narrowness of $T = 0$ resonances, the peak conductance at a finite temperature is reduced by interactions. The valley current depends on both interactions and the device geometry.

References

[1] F.D.M. Haldane, J. Phys. C 14 (1981) 2585.
[2] C.L. Kane, M.P.A. Fisher, Phys. Rev. Lett. 68 (1992) 1220.
[3] J.M. Kinaret et al., Phys. Rev. B (January 15, 1998), to appear.
[4] R. Fazio, F.W.J. Hekking, A.A. Odintsov, Phys. Rev. Lett. 74 (1995) 1843.
[5] For details see A. Mattsson, S. Eggert, J.M. Kinaret, Phys. Rev. B (December 15, 1997), to appear.
[6] D. Loss, Phys. Rev. Lett. 69 (1992) 343.
[7] J. Liu et al., Nature 385 (1997) 780.
[8] S.J. Tans et al., Nature 386 (1997) 474.