Comment on "Néel Order in Doped Quasi-One-Dimensional Antiferromagnets"

In the recent Letter by Eggert, Affleck, and Horton [1], the interesting problem of the influence of nonmagnetic impurities on the Néel temperature of quasi-one-dimensional spin-$\frac{1}{2}$ Heisenberg antiferromagnets (1DHAF) was studied. Nonmagnetic impurities in the system are modeled by a set of spin chains of arbitrary finite lengths L (i.e., each nonmagnetic impurity cuts the 1DHAF), coupled to each other via an interchain (three- or two-dimensional) coupling. The coupling between those chains is treated in the mean-field-like approximation. In this approximation it is necessary to find the general temperature dependence of staggered magnetic susceptibilities of those finite 1DHAF chains. The authors claim that they calculated such a dependence in the framework of the bosonization approximation for general L and T [2]. In this Comment, we point out that such a claim is invalid: The results of the bosonization approximation obtained in [1] are correct only for some ranges of L, T, and the total magnetization, S^z.

The bosonization picture approximates the behavior of low-lying excitations of quantum spin chains (for the 1DHAF those low-lying excitations are spinons [3]) by the one of particle-hole excitations in the vicinity of Fermi point(s) for spinons [4] and zero modes [5]. Operators of particle-hole excitations satisfy bosonic commutation relations if one neglects the behavior of states of spinons in the deep of their Fermi sea and linearizes the dispersion law of spinon about Fermi point(s) [4,5]. The linearization of the dispersion law of a spinon implies that energies, at which the bosonization approximation can be applied (and, hence, considered temperatures), must be at least much smaller than the bandwidth of spinon. Moreover, it is well known that in the 1DHAF bound states of spinons (spin strings) exist [6]. Those bound states should reveal themselves for higher T and cannot be taken into account in the bosonization approximation, in principle. This brings into question the applicability of the bosonization analysis of spin-spin correlation functions of the 1DHAF from [1] for high enough T.

When calculating correlation functions, the authors of [1] essentially use the properties of elliptic theta functions. Some of those elliptic functions were obtained in [1] by summation of zero modes over the *total* range of S^z [cf. Eq. (13) of [1]]. It turns out that for large values of $|S^z|$ one cannot employ the velocity of spinons in the form used by [1]: $v = \pi J/2$. In reality, that velocity depends on S^z and is equal to $\pi J/2$ only at $S^z = 0$. The velocity decreases with $|S^z|$ and approaches zero at $S^z = \pm L/2$. The authors of [1] do not take into account such a decrease, and, hence, their study of correlation functions is invalid at this point. For example, one cannot just simply consider the limit $LT/v \to 0$ as $LT \to 0$, because states with $v \to 0$ were used to get the formula for the correlation function: This limitation procedure needs more careful analysis. Hence, the scaling behavior of correlation functions, assumed in [1] can be right only for some ranges of values of LT and S^z.

Usually the bosonization (conformal) approximate picture of the 1DHAF is used for large L. At least for finite small L one should not replace finite sums by integrals, as [1] did. Also the consideration of the small values of L, and, hence, the set of eigenstates of spinons bounded from below and any values of the Fermi momenta, k_F, comparing to deviations from them, k (the bosonization usually studies $k \ll k_F$ [4,5]), can produce deviations of the commutation relations for particle-hole excitations from the bosonic ones [4]. This questions the derivation of Eq. (12) of [1], which is essentially based on the *bosonic* properties of particle-hole excitations.

According to the above, the scaling, predicted in [1] [cf. Eq. (19) of [1]], should be valid only for small T, small values of S^z (i.e., large v), and large enough values of L. The comparison of the results of analytical bosonization calculations with Monte Carlo numerical simulations cannot convince one that analytical results are valid for general L and T, because the authors used very small temperatures in their numerical simulations, for which the bosonization approximation is certainly valid. Thorough analysis of the applicability of the bosonization approximation for the problem of the Néel temperatures of quasi-one-dimensional antiferromagnetic systems doped with nonmagnetic impurities is still necessary.

A. A. Zvyagin

Institute for Low Temperature Physics
and Engineering of NAS of Ukraine
Kharkov 61164, Ukraine

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[2] In the introduction, the authors mention that the field theory treatment is correct in the asymptotic low T, large L limit, but then they claim to consider the case of general L and T using the field theory approach.
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[5] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[6] See, e.g., M. Takahashi, *Thermodynamics of One-Dimensional Solvable Models* (Cambridge University, Cambridge, 1999).

