Emission Noise in an Interacting Quantum Dot: Role of Inelastic Scattering and Asymmetric Coupling to the Reservoirs

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A theory is developed for the emission noise at frequency ν in a quantum dot in the presence of Coulomb interactions and asymmetric couplings to the reservoirs. We give an analytical expression for the noise in terms of the various transmission amplitudes. Including the inelastic scattering contribution, it can be seen as the analog of the Meir-Wingreen formula for the current. A physical interpretation is given on the basis of the transmission of one electron-hole pair to the concerned reservoir where it emits an energy after recombination. We then treat the interactions by solving the self-consistent equations of motion for the Green functions. The results for the noise derivative versus eV show a zero value until $eV = h\nu$, followed by a Kondo peak in the Kondo regime, in good agreement with recent measurements in carbon nanotube quantum dots.

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In quantum devices, the fluctuations of electrical current provide information on the dynamics of electrons [1–4], as well as on the energy-photon exchange with the measurement circuit or with the electromagnetic environment [5–16]. Understanding the nature of these fluctuations in a quantum dot (QD) is thus a crucial step insofar as this system is the elementary brick of quantum circuits. The measurement of current fluctuations in a QD is becoming more and more precise, and reliable results are now available both at zero frequency [17–19] and finite frequency [20–22]. Interpreting these experimental findings turned out to be a challenging task, especially in the case of a biased interacting QD with asymmetric couplings to the reservoirs. In view of the challenges, there is an increasing need to develop a theory for calculating the current noise in nonequilibrium, incorporating the inelastic scattering contributions that play a crucial role when Coulomb interactions are present. So far, most of the noise calculations in a QD connected to left (L) and right (R) reservoirs either do not distinguish between the noise in the L reservoir and that in the R reservoir [23–31] or assume that the left coupling strength Γ_L and the right coupling strength Γ_R are equal [24,32–34], in apparent contradiction with experiments [20–22]. Indeed, the measured asymmetry of the couplings can be very large, e.g., $a = 11$ [22], where $a = \Gamma_L/\Gamma_R$ is the asymmetry factor. Certainly, there are theoretical works where the distinction between left and right couplings is made, but these works are limited to the calculations of the zero-frequency noise [28] and symmetrized noise (generally not the quantity measured in experiments) both for noninteracting [2,3,35–38] and interacting QDs [39,40]. In some other works, a linear combination of the autocorrelators and of the cross-correlators is calculated [41,42]. In summary, developing an efficient theory to calculate the finite-frequency noise in nonequilibrium, and investigating the effects of Coulomb interactions and of coupling asymmetry on the noise profile in each reservoir are important unsolved issues which we address in this Letter.

The noise considered here is the emission noise [43–45] at frequency ν, $S_{\alpha\beta}(\nu) = \int_{-\infty}^{\infty} \langle \Delta\hat{I}_\alpha(t)\Delta\hat{I}_\beta(0)\rangle e^{-2i\pi\nu t}dt$, where $\Delta\hat{I}_\alpha(t) = \hat{I}_\alpha(t) - \langle\hat{I}_\alpha\rangle$ is the deviation of the current from its average value [the index α (β) represents one of the two reservoirs]. We calculate $S_{\alpha\beta}(\nu)$ in an interacting QD by using the nonequilibrium Keldysh Green function technique. When the system is in a steady state, we establish the following formula:

$$\mathcal{S}_{\alpha\beta}(\nu) = \frac{e^2}{h}\sum_{\gamma\delta}\int_{-\infty}^{\infty} d\varepsilon M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu) f_\gamma^e(\varepsilon) f_\delta^h(\varepsilon - h\nu), \quad (1)$$

where $f_\gamma^e(\varepsilon)$ and $f_\delta^h(\varepsilon) = 1 - f_\delta^e(\varepsilon)$ are the Fermi-Dirac functions for electrons in the γ reservoir and holes in the δ reservoir, respectively, and where the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$ are listed in Table I. These elements are written in terms of the transmission amplitude $t_{\alpha\beta}(\varepsilon)$, the transmission coefficient $\mathcal{T}_{\alpha\beta}(\varepsilon) = |t_{\alpha\beta}(\varepsilon)|^2$, the reflection amplitude $r_{\alpha\alpha}(\varepsilon) = 1 - t_{\alpha\alpha}(\varepsilon)$, and an effective transmission coefficient defined as $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon) = 2\mathrm{Re}\{t_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon)$ [46]. The transmission amplitude is related to the retarded Green function in the QD for spin σ, $G_\sigma^r(\varepsilon)$, through:

TABLE I. Expressions of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$ involved in Eq. (1) for the noise $S_{\alpha\beta}(\nu)$ of an interacting QD with arbitrary coupling symmetry. $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$ is an effective transmission coefficient defined as $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)=2\mathrm{Re}\{t_{\alpha\alpha}(\varepsilon)\}-\mathcal{T}_{\alpha\alpha}(\varepsilon)$.

$M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$	$\gamma=\delta=L$	$\gamma=\delta=R$	$\gamma=L,\ \delta=R$	$\gamma=R,\ \delta=L$
$\alpha=L$ $\beta=L$	$\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon)\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon-h\nu)$ $+\lvert t_{LL}(\varepsilon)-t_{LL}(\varepsilon-h\nu)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$[1-\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon-h\nu)]$
$\alpha=R$ $\beta=R$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon)\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon-h\nu)$ $+\lvert t_{RR}(\varepsilon)-t_{RR}(\varepsilon-h\nu)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon-h\nu)]$	$[1-\mathcal{T}_{LR}^{\mathrm{eff},R}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon-h\nu)$
$\alpha=L$ $\beta=R$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$ $\times[r_{LL}^*(\varepsilon)r_{LL}(\varepsilon-h\nu)-1]$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-h\nu)$ $\times[r_{RR}(\varepsilon)r_{RR}^*(\varepsilon-h\nu)-1]$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-h\nu)$ $\times r_{LL}^*(\varepsilon)r_{RR}^*(\varepsilon-h\nu)$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$ $\times r_{RR}(\varepsilon)r_{LL}(\varepsilon-h\nu)$
$\alpha=R$ $\beta=L$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-h\nu)$ $\times[r_{LL}(\varepsilon)r_{LL}^*(\varepsilon-h\nu)-1]$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$ $\times[r_{RR}^*(\varepsilon)r_{RR}(\varepsilon-h\nu)-1]$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$ $\times r_{LL}(\varepsilon)r_{RR}(\varepsilon-h\nu)$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-h\nu)$ $\times r_{RR}^*(\varepsilon)r_{LL}^*(\varepsilon-h\nu)$

$t_{\alpha\beta}(\varepsilon)=i\sqrt{\Gamma_\alpha\Gamma_\beta}G_\sigma^r(\varepsilon)$, where $\Gamma_\alpha=2\pi\rho_\alpha|V_\alpha|^2$ is the coupling between the QD and the α reservoir, V_α being the electron hopping amplitude between the QD and the α reservoir, the density of states of which is ρ_α. To lighten the notations, we do not put a spin index to $t_{\alpha\beta}(\varepsilon)$ since we consider a spin-unpolarized QD.

It is important to underline that even if the noise formula for the interacting QD shows an apparent similarity to the one obtained for the noninteracting QD [33], the two formulas are distinct since the effective transmission coefficient $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$ differs from $\mathcal{T}_{LR}(\varepsilon)$. Especially $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$ incorporates inelastic scattering contributions [47,48] which are crucial in interacting systems at finite temperature and voltage. In the absence of interactions, or when only elastic scattering processes are present, $\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon)$ simply equals $\mathcal{T}_{LR}(\varepsilon)$: this result follows from the optical theorem which is verified in this case [49], and allows one to recover the formula established in Ref. [33] in the noninteracting case. The difference between the noise formula in Eq. (1) and its noninteracting counterpart can be seen as the exact analog of the difference between the Meir-Wingreen formula for the current valid in the presence of interactions [50] and the Landauer formula obtained using scattering theory.

The proof of Eq. (1) is the following (see Ref. [49] for details): we start from Eqs. (A11)–(A15) of Ref. [33], obtained in the flat wideband limit for the conduction band after having factorized the two-particle Green function in the QD into a product of single-particle Green functions. Provided that the system is in a steady state, we have [51,52]: $G_\sigma^{\gtrless}(\varepsilon)=G_\sigma^r(\varepsilon)\Sigma_{\mathrm{tot},\sigma}^{\gtrless}(\varepsilon)G_\sigma^a(\varepsilon)$, where $\Sigma_{\mathrm{tot},\sigma}^{\gtrless}(\varepsilon)$ is the total self-energy [51,53]: $\Sigma_{\mathrm{tot},\sigma}^{\gtrless}(\varepsilon)=\Sigma_{L,\sigma}^{\gtrless}(\varepsilon)+\Sigma_{R,\sigma}^{\gtrless}(\varepsilon)+\Sigma_{\mathrm{int},\sigma}^{\gtrless}(\varepsilon)$, with $\Sigma_{\alpha,\sigma}^{\gtrless}(\varepsilon)$, the self-energy brought by the coupling with the α reservoir, and $\Sigma_{\mathrm{int},\sigma}^{\gtrless}(\varepsilon)$, the additional self-energy brought by the interactions in the QD. Making use of these relations and noticing that the linear and quadratic terms in $\Sigma_{\mathrm{int},\sigma}^{\gtrless}(\varepsilon)$ cancel in the steady state, one derives Eq. (1).

In the same way that in the Landauer approach the current is interpreted in terms of transmission of electrons from L reservoir to R reservoir, the autocorrelator $\mathcal{S}_{\alpha\alpha}(\nu)$ can be interpreted in terms of transmission of $e-h$ pairs or their constituents through the QD, from all possible initial locations, before the pairs recombine leading to the emission of an energy $h\nu$ in the α reservoir. To get $\mathcal{S}_{\alpha\alpha}(\nu)$, we thus have to identify the whole set of such physical processes for each given initial state, determine their transmission amplitudes t_i, and take the quantum superposition $|\sum_i t_i|^2$ to calculate the transmission probability. The processes contributing to $\mathcal{S}_{LL}(\nu)$ are six in number as depicted in the top row of Fig. 1. We restrict the discussion to $\mathcal{S}_{LL}(\nu)$ because one can straightforwardly deduce $\mathcal{S}_{RR}(\nu)$ by interchanging L and R indices.

When the $e-h$ pair is initially located in the L reservoir, there are three possibilities to emit energy in the L reservoir by recombination of $e-h$ pairs: (i) through process P_1 in which one electron of energy ε (green sphere) and one hole of energy $\varepsilon-h\nu$ (blue sphere) both experience an excursion into the QD and come back to the L reservoir, corresponding to the transmission amplitude $t_1=t_{LL}(\varepsilon)t_{LL}^*(\varepsilon-h\nu)$; (ii) through process P_2 in which the electron experiences an excursion into the QD and comes back to the L reservoir, whereas the hole is reflected by the left barrier, corresponding to the transmission amplitude $t_2=t_{LL}(\varepsilon)r_{LL}^*(\varepsilon-h\nu)$; and (iii) through process P_3 in which the hole experiences an excursion into the QD and comes back to the L reservoir whereas the electron is reflected, corresponding to the transmission amplitude $t_3=r_{LL}(\varepsilon)t_{LL}^*(\varepsilon-h\nu)$. By taking the quantum superposition of these three processes, $|t_1+t_2+t_3|^2$, we get a contribution to the noise which is equal to the matrix element $M_{LL}^{LL}(\varepsilon,\nu)$ of Table I [49]. Note that even if the amplitudes $t_{1,2,3}$ involve the L index only, we use the subscript LR in the notation for the effective transmission coefficient $\mathcal{T}_{LR}^{\mathrm{eff},L}(\varepsilon)$, for the reason that it gives back $\mathcal{T}_{LR}(\varepsilon)$ when the optical theorem holds [49].

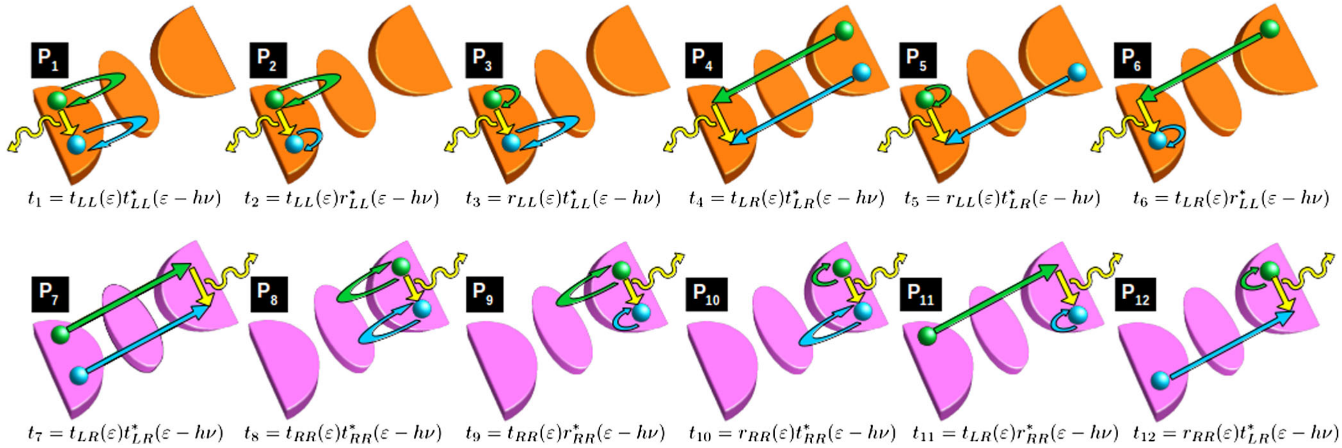

FIG. 1. Illustration of the six physical processes contributing to $\mathcal{S}_{LL}(\nu)$ with the emission of an energy $h\nu$ in the L reservoir (top row with orange background devices), and the other six physical processes contributing to $\mathcal{S}_{RR}(\nu)$ with the emission of an energy $h\nu$ in the R reservoir (bottom row with pink background devices). The transmission amplitude t_i of the $e-h$ pair for each process with $i \in [1, 12]$ is indicated at the bottom of each diagram. A green (blue) sphere represents an electron (a hole) and a yellow wavy arrow represents the emission of energy $h\nu$ in one of the reservoirs.

When the $e-h$ pair is initially located in the R reservoir, both particles cross the entire structure to emit energy in the L reservoir by recombination, as depicted in Fig. 1 (P_4), giving rise to the transmission amplitude $t_4 = t_{LR}(\varepsilon)t^*_{LR}(\varepsilon - h\nu)$, which leads to the matrix element $M^{RR}_{LL}(\varepsilon, \nu)$ of Table I after taking $|t_4|^2$. When the electron is initially located in the L reservoir and the hole in the R reservoir, as depicted in Fig. 1 (P_5), the electron is reflected and the hole transmitted, giving rise to the transmission amplitude $t_5 = r_{LL}(\varepsilon)t^*_{LR}(\varepsilon - h\nu)$ which leads to the matrix element $M^{LR}_{LL}(\varepsilon, \nu)$. By symmetry, the transmission amplitude in process P_6 is $t_6 = t_{LR}(\varepsilon)r^*_{LL}(\varepsilon - h\nu)$, leading to the matrix element $M^{RL}_{LL}(\varepsilon, \nu)$. We do not need to take any quantum superposition for the three processes P_4–P_6 as each of them corresponds to a different initial state.

To get the cross-correlators, one needs to consider the interference terms between the processes accompanied by an emission of energy in both reservoirs [2,3,32]. Our study shows that the sum $\mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)$ corresponds to the interference between the processes P_5 and P_{11} as regards the term proportional to $f^e_L(\varepsilon)f^h_R(\varepsilon - h\nu)$, since $M^{LR}_{LR}(\varepsilon, \nu) + M^{LR}_{RL}(\varepsilon, \nu) = t_5 t^*_{11} + t^*_5 t_{11}$, and to the interference between the processes P_6 and P_{12} as regards the term proportional to $f^e_R(\varepsilon)f^h_L(\varepsilon - h\nu)$, since $M^{RL}_{LR}(\varepsilon, \nu) + M^{RL}_{RL}(\varepsilon, \nu) = t_6 t^*_{12} + t^*_6 t_{12}$. These interference terms can be either positive or negative according to the relative values of ε and ν, but become strictly negative at zero frequency due to charge conservation. As far as the contributions proportional to $f^e_\alpha(\varepsilon)f^h_\alpha(\varepsilon - h\nu)$ are concerned, they are given by the interference between the process P_7 and the set of processes P_1–P_3 when $\alpha = L$, and between the process P_4 and the set of processes P_8–P_{10} when $\alpha = R$.

The noise, given by Eq. (1) with $M^{\delta\gamma}_{\alpha\beta}(\varepsilon, \nu)$ of Table I, is completely determined once the retarded Green function $G^r_\sigma(\varepsilon)$ in the QD is known. For the noninteracting single energy level QD, we take the Breit-Wigner form: $G^r_\sigma(\varepsilon) = [\varepsilon - \varepsilon_0 + i(\Gamma_L + \Gamma_R)/2]^{-1}$ where ε_0 is the QD energy level. For the interacting single energy level QD, we use the self-consistent renormalized equation-of-motion approach, as developed in Refs. [54–56], which applies to both equilibrium and nonequilibrium and allows one to determine $G^r_\sigma(\varepsilon)$ [49]. It has been successfully used [57] to quantitatively explain the experimental results [58] about the interplay of spin accumulation and magnetic field in a Kondo QD, and is well adapted to describe the Kondo regime in which the noise measurements are performed [22].

In Fig. 2, we report the noise derivative $d\mathcal{S}_{\alpha\alpha}(\nu)/dV$ as a function of the voltage V for two values of $a = \Gamma_L/\Gamma_R$ and U (with $\varepsilon_0 = -U/2$), as well as the derivative of the sum of the cross-correlators $d[\mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)]/dV$. For completeness, we also plot the derivative of the total noise, $d\mathcal{S}_{\mathrm{tot}}(\nu)/dV$, where $\mathcal{S}_{\mathrm{tot}}(\nu) = [\mathcal{S}_{LL}(\nu) + a^2\mathcal{S}_{RR}(\nu) - a\mathcal{S}_{LR}(\nu) - a\mathcal{S}_{RL}(\nu)]/(1+a)^2$, following recent theoretical works which show, by using a current conservation argument along with the Ramo-Shockley theorem, that this is the quantity which is measured in experiments [37,38,40]. A common point to all the curves is the presence of a plateau of value zero at voltage smaller than frequency, here $|V| < h\nu/e = 0.32$ mV, since $h\nu = 78$ GHz. The origin of this plateau is related to the fact that the system cannot emit at a frequency higher than the energy provided to it, i.e., the voltage, in full agreement with experiments [21,22]. In the absence of interaction [Figs. 2(a) and 2(b)], the noise derivatives present a broad peak at $|eV| > h\nu$. Its intensity is larger for $d\mathcal{S}_{RR}(\nu)/dV$ than for $d\mathcal{S}_{LL}(\nu)/dV$ for both symmetric and asymmetric couplings, due to the fact that the L reservoir is grounded ($\mu_L = 0$). The effect of the coupling

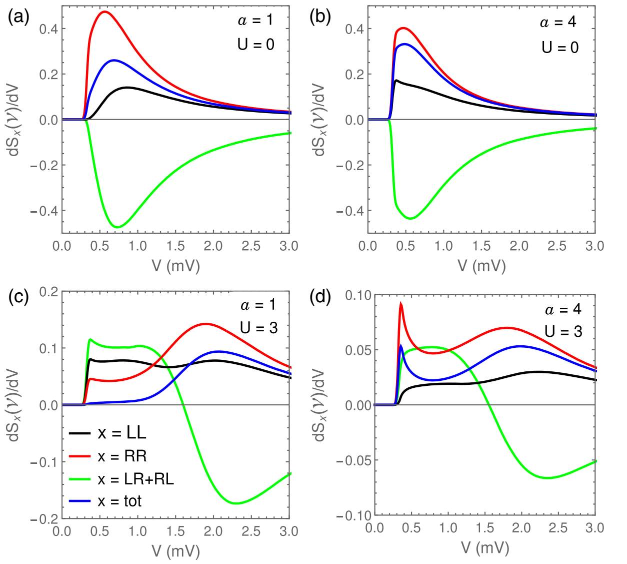

FIG. 2. Noise derivative $d\mathcal{S}_x(\nu)/dV$ as a function of V (with $\mu_L = 0, \mu_R = -eV$) at $T = 80$ mK, $\nu = 78$ GHz (chosen such that $h\nu < k_B T_K$) for $\varepsilon_0 = -U/2$ (middle of the Kondo ridge). (a) and (b): $U = 0$. (c) and (d): $U = 3$ meV. (a) and (c): $\Gamma_{L,R} = 0.5$ meV ($a = 1$). (b) and (d): $\Gamma_L = 0.8$ meV, $\Gamma_R = 0.2$ meV ($a = 4$). A Kondo peak is observed close to $eV = h\nu$ when $U \neq 0$. Plots for $V < 0$ are not shown since $d\mathcal{S}_x(\nu)/dV$ is an odd function in V.

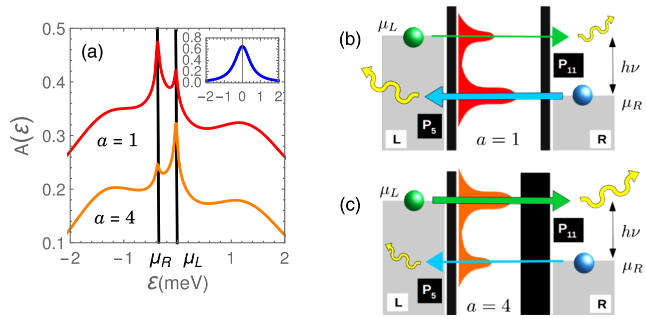

FIG. 3. (a) Spectral density $A(\varepsilon) = -\pi^{-1}\mathrm{Im}\{G^r_\sigma(\varepsilon)\}$ for $\Gamma_{L,R} = 0.5$ meV ($a = 1$), and $\Gamma_L = 0.8$ meV, $\Gamma_R = 0.2$ meV ($a = 4$), at $U = 3$ meV, $\varepsilon_0 = -U/2$, and $T = 80$ mK. The vertical lines indicate the positions of $\mu_R = -eV = -0.35$ mV and $\mu_L = 0$. The $a = 1$ curve has been vertically translated for clarity. The spectral density at $U = 0$ is shown in blue in the inset. (b) and (c) Schematic representation of the relative importance of the transmission processes at low temperature, low transmission, and for $eV \approx h\nu$: P_5 is dominant over P_{11} for $a = 1$, and P_{11} is dominant over P_5 for $a = 4$.

asymmetry is to shift the position of the broad peak towards lower values of V. Note that in both cases, the derivative of $\mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)$ is sign negative [green curves in Figs. 2(a) and 2(b)]. In the presence of interactions, the electronic transport through a QD is strongly affected. In the Kondo regime, when the number of electrons in the QD is equal to 1 and $T \ll T_K$ (T_K being the Kondo temperature), the differential conductance shows a Kondo peak around $V = 0$, in addition to the broad peaks resulting from the Coulomb blockade [49]. These effects have their counterparts in the noise. Indeed, the noise derivative shows two clear features [Figs. 2(c) and 2(d)]: a Kondo peak above $|eV| = h\nu$, and a secondary broad peak in the proximity of $|eV| = U/2$, corresponding to the boundaries of the Coulomb blockade structure, in full agreement with experiments [22]. We observe that the noise intensity is reduced in the presence of interactions, as expected in the Kondo regime [24]. This is related to the fact that $\mathcal{T}^{\mathrm{eff},\alpha}_{LR}(\varepsilon) > \mathcal{T}_{LR}(\varepsilon)$ (curves not shown), leading to a decrease of $M^{LR}_{\alpha\alpha}(\varepsilon,\nu)$ which provides the dominant contribution at low temperature. Moreover, the derivative of $\mathcal{S}_{LR}(\nu) + \mathcal{S}_{RL}(\nu)$ changes sign at $|eV| = U/2$, going from positive to negative values with increasing V. It explains why for symmetric couplings ($a = 1$) the total noise derivative becomes smaller below $|eV| = U/2$. We also notice that the height of the Kondo peak in $d\mathcal{S}_{LL}(\nu)/dV$ is larger than in $d\mathcal{S}_{RR}(\nu)/dV$ when $a = 1$. This relative order in magnitude is reversed when $\Gamma_L > \Gamma_R$: in this case the Kondo peak becomes more prominent for the more weakly coupled reservoir. The explanation is the following: when $a \neq 1$, (i) the more pronounced Kondo resonance in the density of states is pinned at the chemical potential (μ_L) of the more strongly coupled reservoir [orange curve in Fig. 3(a)], and (ii) at low temperature, the main process contributing to $d\mathcal{S}_{LL}(\nu)/dV$ is P_5 with a probability equal to $\mathcal{T}_{LR}(\varepsilon - h\nu)$ at low transmission, whereas the main process contributing to $d\mathcal{S}_{RR}(\nu)/dV$ is P_{11} with a probability equal to $\mathcal{T}_{LR}(\varepsilon)$ at low transmission. In the process P_5 of Fig. 3(c), there is a transfer of holes from the R reservoir to the L reservoir at energy close to $\mu_R = -eV$, in the vicinity of which a relatively smaller Kondo resonance is observed, whereas in the process P_{11} of Fig. 3(c), there is a transfer of electrons from the L reservoir to the R reservoir at energy close to $\mu_L = 0$, in the vicinity of which a stronger Kondo resonance is observed [orange curve in Fig. 3(a)]. Since P_{11} contributes to $d\mathcal{S}_{RR}(\nu)/dV$, the Kondo peak is more visible in $d\mathcal{S}_{RR}(\nu)/dV$. In the same way, Fig. 3(b) illustrates how the height of the Kondo peak in $d\mathcal{S}_{LL}(\nu)/dV$ is larger than that in $d\mathcal{S}_{RR}(\nu)/dV$ when $a = 1$.

We have established a general formula for the emission noise in an interacting QD asymmetrically coupled to reservoirs taking the inelastic scattering contributions into account, and we have given a physical interpretation of the results in terms of the transmission of an $e - h$ pair through the QD with an emission of energy. Combining the theory with the equation-of-motion approach to determine the transmission amplitudes entering the noise formula, we have discussed the profile of the noise derivative. The obtained results explain most of the distinctive features recently observed for the noise in a carbon nanotube QD, specially, the presence or the absence of a narrow peak in $d\mathcal{S}_x(\nu)/dV$ versus V in the vicinity of $\pm h\nu/e$, and why the Kondo peak in the noise derivative is more prominent in the more weakly coupled reservoir. The theory developed in this Letter can be applied to treat other realistic systems.

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