Shot noise in Weyl semimetals

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(Received 16 May 2017; revised manuscript received 20 August 2017; published 4 October 2017)

We study the effect of inelastic processes on the magnetotransport of a quasi-one-dimensional Weyl semimetal, using a modified Boltzmann-Langevin approach. The magnetic field drives a crossover to a ballistic regime in which the propagation along the wire is dominated by the chiral anomaly, and the role of fluctuations inside the sample is exponentially suppressed. We show that inelastic collisions modify the parametric dependence of the current fluctuations on the magnetic field. By measuring shot noise as a function of a magnetic field, for different applied voltage, one can estimate the electron-electron inelastic length $l_{\rm ee}$.

DOI: 10.1103/PhysRevB.96.165406

I. INTRODUCTION

Weyl semimetals have been a subject of active experimental and theoretical research due to their unusual transport properties [1–13]. A Weyl semimetal is characterized by a three-dimensional band structure, where valence and conduction bands touch at discrete isolated points in the Brillouin zone. Excitations in the vicinity of the band degeneracy points are governed by the Weyl Hamiltonian $H = \pm\hbar\,\mathbf{p}\cdot\sigma$.

The nontrivial topology of a Weyl semimetal can be revealed by applying an external magnetic field, which leads to the formation of Landau levels (LL). The Weyl nodes result in the emergence of chiral zero Landau levels, protected against scattering for a sufficiently smooth disorder. Remarkably, transport properties in the presence of a magnetic field can be described by the semiclassical Boltzmann equation [14–16]

$$\frac{\partial f(\mathbf{p},\mathbf{r})}{\partial t} + \dot{\mathbf{r}}\frac{\partial f(\mathbf{p},\mathbf{r})}{\partial r} + \dot{\mathbf{p}}\frac{\partial f(\mathbf{p},\mathbf{r})}{\partial \mathbf{p}} = I[f(\mathbf{p},\mathbf{r})]. \tag{1}$$

Here $f(\mathbf{p})$ is the distribution function, $\mathbf{p}$ is the momentum, $I[f(\mathbf{p})]$ is the collision integral, and

$$\begin{aligned}\dot{\mathbf{r}} &= \frac{\partial\epsilon_p}{\partial\mathbf{p}} + \dot{\mathbf{p}}\times\Omega_{\mathbf{p}},\\ \dot{\mathbf{p}} &= e\mathbf{E} + \dot{\mathbf{r}}\times e\mathbf{B}.\end{aligned} \tag{2}$$

The nontrivial topology is reflected in Berry curvature terms that appear in the Liouville operator [17–19]

$$\begin{aligned}\Omega_{\mathbf{p}} &= \nabla_{\mathbf{p}}\times\mathbf{A}_{\mathbf{p}},\\ \mathbf{A}_{\mathbf{p}} &= i\langle u_{\mathbf{p}}|\nabla_{\mathbf{p}}u_{\mathbf{p}}\rangle,\end{aligned} \tag{3}$$

where $u_{\mathbf{p}}$ is a periodic part of the Bloch wave function. These terms originate from the chiral anomaly, and are absent in topologically trivial matter. They give rise to ballistic propagation [16] and nonlocal ac conductance [20].

In this work we study the effect of inelastic processes on the magnetotransport of a quasi-one-dimensional Weyl semimetal, within a semiclassical description. The wire, with cross-section A, is pierced by a magnetic field B, oriented along the wire in $\hat{z}$ direction. We assume that the internodal scattering length is much longer than the system length L. This allows us to focus in the vicinity of a single Weyl node with a given chirality. The final answer is given by a sum over all Weyl nodes.

We first note that similar to a metal, the conductance is unaffected by interactions between electrons, as those do not lead to momentum relaxation. Moreover, as the conductance is measured as a response to the difference of total electrochemical potentials, the results obtained for interacting electrons within self-consistent field approximation, and for noninteracting electrons subjected to a difference in chemical potential, coincide. Consequently, the conductance can be calculated in the limit of noninteraction Weyl fermions, neglecting both the inelastic collision integral and the self-consistent electric field.

Using Eq. (2) for noninteracting Weyl fermions of positive chirality, the density of electric current is given by

$$\mathbf{j}(\mathbf{r}) = \int\frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\left[\frac{\partial\epsilon_p}{\partial\mathbf{p}} + \frac{e\mathbf{B}}{c}\Omega_p\cdot\frac{\partial\epsilon_p}{\partial\mathbf{p}}\right]f(\mathbf{p},\mathbf{r}). \tag{4}$$

Assuming that intranodal scattering is short ($l_{\rm intra}\ll L$), we perform a diffusive approximation $f(\mathbf{p},\mathbf{r}) = \mathbf{f}(\epsilon_{\mathbf{p}},\mathbf{z}) + (\hat{\mathbf{z}}\cdot\mathbf{p})\frac{\tau}{\mathbf{m}}\frac{\partial\mathbf{f}(\epsilon_{\mathbf{p}},\mathbf{z})}{\partial\mathbf{z}}$, where τ is a momentum relaxation (transport) time and m is an effective mass of an electron. The current density within diffusion approximation is

$$\mathbf{j}(z) = \left(D\partial_z\rho - \frac{eB}{4\pi^2\nu}\rho\right), \tag{5}$$

where the density of electrons

$$\rho(z) = \nu\int_{-\infty}^{\infty}d\epsilon\, f(\epsilon,z). \tag{6}$$

Using the fact that the current density is subject to the continuity equation, which in the static limit reads $\partial_z\mathbf{j}(z) = 0$, we find that the electron density obeys a drift-diffusion equation [14–16] characteristic of a one-dimensional random walk with nonequal probabilities:

$$\xi\,\partial_z^2\rho(z) \mp N_\phi\partial_z\rho(z) = 0. \tag{7}$$

Here $\mp$ signs correspond to Weyl nodes of opposite chiralities, ξ is the localization length ($\xi = 2\pi\nu DA$, with ν

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three-dimensional density of states and D diffusion constant), and

$$N_\phi = \frac{AeB}{\Phi_0} = \frac{A}{2\pi}\frac{eB}{\hbar c} \tag{8}$$

is the number of magnetic flux quanta piercing the wire, which also marks the imbalance between left and right moving modes.

The validity of Eqs. (5) and (7) was recently established with Keldysh [21] and supersymmetry [22,23] nonlinear sigma model formalism. Equation (7) shows that while the disorder scattering between different LL disrupts the ballistic propagation of the zero Landau level, the imbalance between the number of left and right moving modes (for a given Weyl node) remains. Equation (7) should be supplemented with the boundary conditions for the value of distribution function at the leads. For the sample subject to voltage difference, the boundary conditions are $f(z=0) = f_F(\epsilon - eV/2), f(z = L) = f_F(\epsilon + eV/2)$; here $f_F(\epsilon)$ is Fermi-Dirac distribution function.

Solving Eq. (7) for the electron density and using Eq. (5) we obtain the conductance

$$G(B) = N_W \frac{e^2 N_\phi}{4\pi\hbar}\coth\left(\frac{L}{2a}\right), \tag{9}$$

in agreement with Keldysh [21] and supersymmetry sigma model calculations [22,23]. Here the drift length $a \equiv \xi/N_\phi = 2\pi\nu D/eB$, and we have multiplied the result by the total number of Weyl nodes N_W. The semiclassical description (7) shows that the magnetic field drives a crossover from diffusive propagation to a ballistic regime, where only the chiral channels contribute to transport, leading in particular, to a positive magnetoconductance [24–27].

We note that while the semiclassical approximation restricts the strength of magnetic fields $\omega_c\tau \ll 1$, the value of L/a may be large provided that $L > E_F\tau l$, where l is an elastic mean free path and E_F is Fermi velocity. Moreover, in order to ignore localization effects, the sample should be shorter than localization length $L/\xi \ll 1$. Both conditions can be fulfilled simultaneously, provided $Ap_F^2 \gg E_F\tau$. In the opposite limit of ultrastrong magnetic field $\omega_c\tau \gg 1$, there is no mixing of Landau levels and we arrive to the Landauer picture of ballistic noiseless transport in N_ϕ channels and the conductance is $G(B) = \frac{2e^2}{h}N_\phi$.

II. FLUCTUATIONS IN DISORDERED WEYL SEMIMETALS

We now turn to a computation of current noise in Weyl semimetals. When calculating the low frequency noise, the self-consistent electric field can be neglected. This is because it leads to the replacement of the inverse Thouless time with the inverse Maxwell time as a characteristic frequency below which the zero frequency limit is achieved. The inelastic collisions on the other hand play an important role for nonequilibrium current fluctuations, and the corresponding collision integrals need to be restored in the kinetic equation (1). Within the diffusive approximation, the latter reads

$$\xi\partial_z^2 f(\epsilon,z) \mp N_\phi\partial_z f(\epsilon,z) - I_{\text{e-e}}[f] - I_{\text{e-ph}}[f] = 0, \tag{10}$$

where $I_{\text{e-e}}$ and $I_{\text{e-ph}}$ are electron-electron and electron-phonon collision integrals.

For topologically trivial metals, current fluctuations can be computed following a Boltzmann-Langevin approach [28,29]. This approach relates the fluctuation of observable quantities, such as current, to the fluctuation of the occupation in the phase space $\delta f(\mathbf{p},\mathbf{r},t)$, with the same applicability as the kinetic equation. We employ this approach, taking into account effects of topology present in Weyl semimetals.

Similar to the current density (4), the density of current fluctuations in a Weyl semimetal can be related to the fluctuation of phase space density as

$$\delta\mathbf{j}(t) = \sum_{\mathbf{p}}\left[\frac{\partial\epsilon_{\mathbf{p}}}{\partial\mathbf{p}}\delta f(\mathbf{p},\mathbf{r},t) + \frac{eB}{4\pi^2\nu}\delta f(\mathbf{p},\mathbf{r},t)\right]. \tag{11}$$

The first term in Eq. (11) is affected by the antisymmetric part of the momentum-dependent distribution function, while the second term is proportional to its symmetric part, i.e., the total density of electrons at a given point. This second term is absent in topologically trivial metals and corresponds to the drift current associated with electrons occupying the zero Landau level.

We note that due to particle number conservation, the correlation function of current fluctuation is independent of the lateral position z. This allows us to compute the correlation of integrated currents:

$$\delta I(t) = \frac{1}{L}\int d^3\mathbf{r}\,\delta\mathbf{j}(\mathbf{r},t). \tag{12}$$

The correlation function of current fluctuation is calculated by representing the kinetic theory of fluctuations through a Keldysh field theory [30]. This approach is identical to the original Boltzmann-Langevin equation for pair-correlation function but allows us to compute current cumulants of any order. While our main focus in this paper is on the pair-correlation function only, the calculation presented hereafter enables one to obtain the full kinetic theory of fluctuations in Weyl semimetals.

The generating function of counting statistics [31] can be written as [32–34]

$$\kappa[\lambda] = \int Df\,D\delta\bar{f}\,e^{i\mathcal{S}[f,\delta\bar{f},\lambda] + i\int dt dz \lambda(t)\delta I(z,t)}. \tag{13}$$

Here the density of particles in the phase space is treated as a field $f \equiv f(\mathbf{p},\mathbf{r},t)$, and the field $\delta\bar{f} \equiv \delta\bar{f}(\mathbf{p},\mathbf{r},t)$ is its conjugate. The field $f(\mathbf{p},\mathbf{r},t)$ consists of a mean value, which solves the Boltzmann equation (10), and a fluctuation part $\delta f(\mathbf{p},\mathbf{r},t)$. The auxiliary counting field λ is chosen in accordance with the correlation function in question. The effective action consists of two parts:

$$\mathcal{S}[f,\delta\bar{f},\lambda] = \mathcal{S}_{\text{Dyn}} + \mathcal{S}_{\text{Noise}}[\lambda]. \tag{14}$$

The dynamical part of the action

$$i\mathcal{S}_{\text{Dyn}} = 2\pi i\nu A\int dt\,d\epsilon\,dz\bigg[\delta\bar{f}(\epsilon,z,t)\bigg\{\frac{\partial}{\partial t} - D\frac{\partial^2}{\partial z^2} + \frac{eB}{4\pi^2\nu}\frac{\partial}{\partial z} + \hat{I}_{\text{e-ph}} + \hat{I}_{\text{e-e}}\bigg\}f(\epsilon,z,t)\bigg] \tag{15}$$

describes the evolution of the particle occupation in the phase space. The cascade creation [35] of the noise is encoded in

$$i\mathcal{S}_{\text{Noise}}[\lambda] = -\nu A \int dt\, d\epsilon\, dz \left(\lambda + \frac{\partial \delta \bar{f}(\epsilon,z,t)}{\partial z}\right)^2 \times f(\epsilon,z,t)[1 - f(\epsilon,z,t)]. \tag{16}$$

There are two modifications of the action in Eq. (14) as compared with the one for normal metals [32–34]. First, the third term (in curly braces) in Eq. (15) is absent in normal metals. This term dictates that the dynamics of noise propagation on scales longer than the elastic collision length is of drift-diffusion type, as opposed to diffusion in normal metals. The second is more subtle and is related to the fluctuating current that couples to the source term in Eq. (13). In normal metals $\delta\mathbf{j}$ is proportional to the velocity $\mathbf{v_p}$ [see the first term in Eq. (11)], which is related to spatial gradients of the density within the diffusive approximation. For this reason, in topologically trivial metals it contributes to the mean value of the current only which is determined by gradients of the density, see Eq. (5), and plays no role in its fluctuations, as the source term in Eq. (13) couples to the integrated current. In Weyl semimetals, on the other hand, this term has a drift contribution, encoded in the second term in Eq. (11), which has no gradients. For that reason it contributes to the fluctuations of currents.

Using the action (14) one expresses the current fluctuations

$$S_2 = \int dt\, \langle \delta I(t) \delta I(0) \rangle \tag{17}$$

through the correlation functions of phase space density operators

$$S_2 = \frac{e^2}{L^2} \int d1 d2 \bigg\{ \delta(1-2) f(1)[1 - f(1)] - \frac{\xi^2}{2}\Big[a^{-2} D^K(1,2) + 4a^{-1} D^R(1,2) \partial_{z_2} f(2)[1 - f(2)]\Big]\bigg\}. \tag{18}$$

Where we have used $1 \leftrightarrow (\epsilon_1, z_1)$, $d1 \leftrightarrow d\epsilon_1 dz_1$ and similarly for 2, for brevity; $D^{K/R}$ are Keldysh and retarded components of δf correlation functions. Computing the correlation functions, in the presence of inelastic scattering (see the Appendix for the details) one finds the low-frequency noise spectrum

$$S_2 = \frac{e^2 \xi}{a^2} \int_0^L dz\, T(z) \frac{\exp[2(L-z)/a]}{(e^{L/a} - 1)^2}. \tag{19}$$

The information about nonequilibrium state of the system is encoded in effective temperature

$$T(z) = \int_{-\infty}^{\infty} d\epsilon\, f(\epsilon,z)[1 - f(\epsilon,z)], \tag{20}$$

which accounts for the spread of the distribution function $f(\epsilon,z)$ determined by Eq. (10) in the presence of all scattering processes.

Equation (19) is the central result of this work. It extends the standard expression for the noise spectrum in normal metals to a Weyl semimetal. While in normal metals the noise is proportional to an integral over effective temperature alone, for a Weyl semimetal it acquires an additional kernel, which depends exponentially on the lateral distance from the leads. The physical reason for that has to do with the chiral anomaly. In normal matter, fluctuations of phase space distribution that occur anywhere in the sample diffusively propagate to the contacts, giving rise to a current noise. For a Weyl semimetal, on the other hand, on scales much greater than a, the fluctuations in the phase-space occupation move predominantly in a ballistic fashion. Hence, in long samples, $L \gg a$, where the transport is dominated by ballistic (deterministic) propagation, stochastic processes are exponentially suppressed, and the system is noiseless.

We now analyze the noise for a number of limiting cases and start from the limit of large inelastic length.

A. Elastic scattering

At equilibrium, the distribution function is of Fermi-Dirac form and Eq. (19) yields

$$S_2 = 2T\, G(B), \tag{21}$$

with $G(B)$ from (9), in accordance with the fluctuation-dissipation theorem. In fact, this result holds even in the presence of inelastic scattering, as can be easily checked by imposing thermal equilibrium, which implies a uniform temperature $T(z) = T$ in Eq. (19).

Next, we study the shot noise in the limit $T = 0$, and finite bias V. Substituting Eqs. (A3) for noninteracting electrons into Eq. (18) one arrives at Eq. (19) that yields

$$S_2 = \frac{e^3 N_W N_\phi V}{16\pi} \frac{\sinh x - x}{\sinh^4(x/2)}, \tag{22}$$

with $x = L/a$ in agreement with [21,23]. It corresponds to Fano factor ($F = S_2/eI$)

$$F_{\text{el}}(x) = \frac{\sinh x - x}{2 \sinh x \sinh^2(x/2)}. \tag{23}$$

At short distances $L \ll a$, the Fano factor approaches the value of diffusive metals 1/3, while it is exponentially suppressed for $L \gg a$, see Fig. 1. This suppression indicates that at large sample sizes (or large magnetic fields) $L \gg a$, all electronic motion is predominantly ballistic.

In our analysis we assume that intranode scattering rate is much larger than internode scattering, such that the scattering

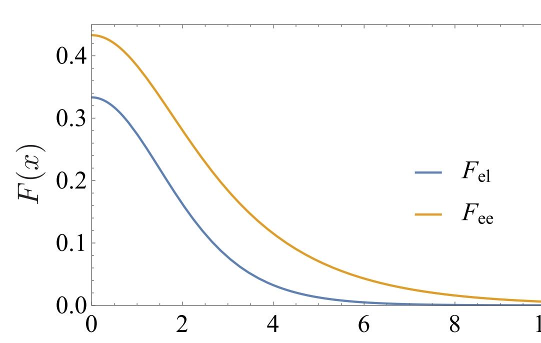

FIG. 1. Fano factors for elastic (F_{el} lower) and inelastic (F_{ee} upper) regimes as functions of the ratio $x = L/a$ of the system length L to the drift length $a = 2\pi\nu D/eB$.

between different Weyl nodes can be neglected. In the opposite limit, for the system longer than the internode relaxation length ($L \gg l_{\text{inter}}$), the topological effects are suppressed, and Fano factor approaches the value $F = 1/3$. The crossover between these regimes for noninteracting electrons was addressed in Ref. [21].

B. Electron-phonon relaxation ($L \gg l_{\text{e-ph}}$)

We now consider the case when the sample is much longer that electron-phonon inelastic length, yet momentum relaxation is still governed by static disorder, namely, $l_{\text{dis}} \ll l_{\text{e-ph}} \ll L$. For electron-phonon inelastic scattering the only conserved quantity is the total density of electron. This implies

$$\int d\epsilon\, I_{\text{e-ph}}[f] = 0. \tag{24}$$

For systems longer that electron-phonon inelastic length the distribution function takes the following form:

$$f(\epsilon,z) = f_F\left(\frac{\epsilon - \mu(z)}{T}\right), \tag{25}$$

where the temperature T is equal to the temperature of phonon bath, and the chemical potential satisfies

$$\left(-\partial_z^2 + a^{-1}\partial_z\right)\mu(z) = 0. \tag{26}$$

For distribution (25) Eq. (19) yields

$$S_2 = 2T\, G(B), \tag{27}$$

corresponding to Nyquist noise with phonon temperature. As in normal metals they are not affected by external bias. Moreover, the Fano factor is given by its value in normal metals $F_{\text{e-ph}} = \frac{2T}{eV}$, and the drift term plays no role.

C. Electron-electron relaxation ($L, a \gg l_{\text{e-e}}$)

For a system longer than the electron-electron collision length and with no electron-phonon scattering there are two propagating modes: density and temperature fluctuations. The latter arise due to the energy conservation for electron-electron scattering

$$\int d\epsilon\, \epsilon\, I_{\text{e-e}}[f] = 0. \tag{28}$$

This implies that the mean value of the distribution function has a form

$$f(\epsilon,z) = f_F\left(\frac{\epsilon - \mu(z)}{T(z)}\right). \tag{29}$$

Plugging this into Eq. (10) one finds

$$\begin{aligned} &\left(-\partial_z^2 + a^{-1}\partial_z\right)\mu(z) = 0,\\ &\left(-\partial_z^2 + a^{-1}\partial_z\right)\left(\frac{1}{2}\mu^2(z) + \frac{\pi^2}{6}T^2(z)\right) = 0. \end{aligned} \tag{30}$$

Solving these equations one finds the chemical potential

$$\mu(z) = \mu + \frac{eV}{2}\frac{1 + e^{L/a} - 2e^{z/a}}{1 - e^{L/a}}, \tag{31}$$

and the effective temperature

$$T^2(z) = \frac{3e^2V^2}{\pi^2}\frac{(e^{L/a} - e^{z/a})(e^{z/a} - 1)}{(e^{L/a} - 1)^2}, \tag{32}$$

here we assumed $T = 0$ at the leads. Substituting these expressions into the distribution function, we obtain

$$S_2 = \frac{\sqrt{3}}{8\pi}\frac{e^3N_W\xi V}{L}\frac{x}{\sinh(x/2)} = \frac{\sqrt{3}}{8\pi}\frac{e^3N_WN_\phi V}{\sinh(x/2)}, \tag{33}$$

and the corresponding Fano factor

$$F_{\text{ee}}(x) = \frac{\sqrt{3}}{4\cosh x/2}. \tag{34}$$

Note, that unlike normal metal, where different inelastic regimes result in different numerical values of the Fano factors, in Weyl semimetals different inelastic processes have different magnetic field dependence. For $L \ll a$, $S_2 \simeq \frac{\sqrt{3}}{4}eI$ as expected for normal metals [36,37]. In the limit $L \gg a$ the shot noise is exponentially suppressed. By changing the magnetic field one should be able to experimentally explore the crossover from diffusive to "ballistic" regimes. Moreover, the different parametric dependence in Eqs. (23) and (34) allows us to estimate l_{ee} from measurement of shot noise as a function of magnetic field. To perform this estimate one should fix the value of V, T, such that $l_{\text{ee}} < L$ and scan the Fano factor by changing the magnetic field. At low values of magnetic fields in this regime the Fano factor will be approximately equal to F_{ee} that will crossover into F_{el} with increasing B at $a \sim l_{\text{ee}}$.

III. CONCLUSIONS

In this paper we studied the current noise in Weyl semimetals in the presence of inelastic scattering. We constructed a Boltzmann-Langevin approach, taking into account chiral anomaly effects. Similar to a case of elastic propagation [21,23], we find that chiral anomaly effects dominate the evolution of the occupation in phase space for samples longer than the drift length $a \equiv \xi/N_\phi = 2\pi\nu D/eB$, experimentally controllable by tuning the magnetic field. This gives rise to deterministic (ballistic) propagation of fluctuations at long samples $L \gg a$, in a direction determined by the chirality. As a result, only stochastic processes that occur within a distance a from the leads, contribute to the current noise.

We show that inelastic collisions resulting from electron-electron interactions, modify this parametric dependence on the magnetic field. By measuring shot noise as a function of a magnetic field, for different applied voltage, which interpolates between the elastic and inelastic limit, one can estimate the electron-electron inelastic length l_{ee}.

Finally, we show that as in normal metals, the presence of electron-phonon scattering suppresses the shot noise, and the current fluctuations correspond to Nyquist noise, governed by the phonon bath temperature.

ACKNOWLEDGMENTS

This work was supported by ISF (Grant No. 584/14), Israeli Ministry of Science, Technology and Space, and by RFBR Grant No. 15-52-06009. D.M. acknowledges support from the Israel Science Foundation (Grant No. 737/14) and from the European Union's Seventh Framework Programme (FP7/2007-2013) under Grant No. 631064. P.G.M. was in part supported by the Deutsche Forschungsgemeinschaft (DFG) via the research center SFB/TR 185. The discussions with D. Bagrets, P. Ostrovsky, and A. Stern are gratefully acknowledged.

APPENDIX: DERIVATION OF EQ. (19)

The action (14) allows us to compute the correlation functions D^α with $\alpha \in \{R,A,K\}$. In the static limit the retarded propagator

$$\left[\xi\left(-\partial_z^2 + a^{-1}\partial_z\right) + I_{\text{e-e}} + I_{\text{e-ph}}\right] D^R(\epsilon,z;\epsilon',z') = \delta(\epsilon-\epsilon')\delta(z-z'). \tag{A1}$$

The advanced propagator D^A is given by Hermitian conjugation of D^R, understood as a matrix, with respect to energy and spatial coordinates. The Keldysh part of the propagator

$$D^K(\epsilon,z;\epsilon',z') = -2\xi \int_0^L dz_1 \int_{-\infty}^{\infty} d\epsilon_1 D^R(\epsilon,z;\epsilon_1,z_1)\partial_{z_1} f(\epsilon_1,z_1) \times [1 - f(\epsilon_1,z_1)]\partial_{z_1} D^A(\epsilon_1,z_1;\epsilon',z'). \tag{A2}$$

The key observation is that diffusion propagator that enters into Eq. (18) contains diffusion propagators integrated over energy $\int d\epsilon\, D^R(\epsilon,z;\epsilon',z') = \mathbf{D}^{\mathbf{R}}(z,z')$ and $\int d\epsilon' D^A(\epsilon,z;\epsilon',z') = \mathbf{D}^A(z,z')$. It follows from particle conservation preserved by both electron-electron and electron-phonon scattering that $\int d\epsilon\, I_{\text{ee}} = \int d\epsilon\, I_{\text{e-ph}} = 0$. This implies that propagators integrated over energy are equal to those in elastic case. Hence the structure of Eq. (19) is preserved in the case of inelastic scattering. We now prove it with more details in some important limiting cases.

1. $l_{\text{ee}}, l_{\text{e-ph}} \gg L$

For the system shorter than the shortest inelastic length the electron-phonon and electron-electron collisions can be neglected. In this case the evolution is purely elastic propagation, and the energy is conserved:

$$D^\alpha(\epsilon,z;\epsilon',z') = \delta(\epsilon-\epsilon')\mathbf{D}^\alpha_\epsilon(z,z'). \tag{A3}$$

Here $\alpha \in \{R,A,K\}$ and we define

$$\mathbf{D}^R(z,z') = \frac{a}{\xi}\frac{1}{1-e^{L/a}}\begin{cases}(e^{z/a}-1)(e^{(L-z')/a}-1), & z<z',\\ (e^{z/a}-e^{L/a})(e^{-z'/a}-1), & z>z'.\end{cases} \tag{A4}$$

The advanced propagator $\mathbf{D}^A(z,z')$ is obtained from $\mathbf{D}^R(z,z')$ by replacing $a \to -a$:

$$\mathbf{D}^K_\epsilon(z,z') = -2\xi \int_0^L dz_1\, \mathbf{D}^R(z,z_1)\partial_{z_1} f(\epsilon,z_1) \times [1 - f(\epsilon,z_1)]\partial_{z_1}\mathbf{D}^A(z_1,z').$$

Using Eq. (A4) and integrating over energy and coordinate we come to Eq. (19).

2. $l_{\text{e-ph}} \ll L$

The diffusion propagator is

$$D^R(\epsilon,z;\epsilon',z') = \partial_\mu f\left(\frac{\epsilon-\mu(z)}{T}\right)\mathbf{D}^R(z,z'),$$
$$D^A(\epsilon,z;\epsilon',z') = \mathbf{D}^A(z,z')\partial_\mu f\left(\frac{\epsilon'-\mu(z')}{T}\right), \tag{A5}$$

and D^K can be expressed through $D^{R/A}$ via (A2). Note that derivatives of distribution function over energies always appear on the outer part of diffusion propagator. The corresponding integral over energies can be easily performed, reproducing Eq. (19).

3. $l_{\text{ee}} \ll L$ and $l_{\text{e-ph}} \gg L$

In case of strong inelastic electron-electron scattering the propagators are

$$D^R(\epsilon,z;\epsilon',z') = \partial_\mu f_F\left(\frac{\epsilon-\mu(z)}{T(z)}\right) \times \left[1 + \frac{3}{\pi^2}\frac{\epsilon-\mu(z)}{T(z)}\frac{\epsilon'-\mu(z')}{T(z')}\right]\mathbf{D}^R(z,z'),$$
$$D^A(\epsilon,z;\epsilon',z') = \left[1 + \frac{3}{\pi^2}\frac{\epsilon-\mu(z)}{T(z)}\frac{\epsilon'-\mu(z')}{T(z')}\right]\mathbf{D}^R \times (z,z')\partial_\mu f_F\left(\frac{\epsilon'-\mu(z')}{T(z')}\right). \tag{A6}$$

The Keldysh part of the propagator is determined via (A2). Using the identities

$$\int_{-\infty}^{\infty}\epsilon\frac{\partial f_F(\epsilon)}{\partial\epsilon}d\epsilon = 0,$$
$$\int_{-\infty}^{\infty}\epsilon\frac{\partial f_F(\epsilon)}{\partial\epsilon} f_F(\epsilon)[1-f_F(\epsilon)]\,d\epsilon = 0, \tag{A7}$$

one reduces Eq. (18) to Eq. (19).

[1] A. M. Turner and A. Vishwanath, Beyond band insulators: Topology of semi-metals and interacting phases, arXiv:1301.0330.

[2] X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates, Phys. Rev. B **83**, 205101 (2011).

[3] A. A. Burkov and L. Balents, Weyl Semimetal in a Topological Insulator Multilayer, Phys. Rev. Lett. **107**, 127205 (2011).

[4] P. Hosur and X. Qi, Recent developments in transport phenomena in Weyl semimetals, C. R. Phys. **14**, 857 (2013).

[5] W. Witczak-Krempa and Y. B. Kim, Topological and magnetic phases of interacting electrons in the pyrochlore iridates, Phys. Rev. B **85**, 045124 (2012).

[6] G. Xu, H. Weng, Z. Wang, X. Dai, and Z. Fang, Chern Semimetal and the Quantized Anomalous Hall Effect in $HgCr_2Se_4$, Phys. Rev. Lett. **107**, 186806 (2011).

[7] H. Weng, C. Fang, Z. Fang, B. A. Bernevig, and X. Dai, Weyl Semimetal Phase in Noncentrosymmetric Transition-Metal Monophosphides, Phys. Rev. X **5**, 011029 (2015).

[8] S.-M. Huang *et al.*, A Weyl fermion semimetal with surface Fermi arcs in the transition metal monopnictide TaAs class, Nat. Commun. **6**, 7373 (2015).

[9] M. Neupane *et al.*, Observation of a three dimensional topological Dirac semimetal phase in high-mobility Cd_3As_2, Nat. Commun. **5**, 3786 (2014).

[10] Z. K. Liu *et al.*, A stable three-dimensional topological dirac semimetal Cd_3As_2, Nat. Mater. **13**, 677 (2014).

[11] S. Y. Xu *et al.*, Discovery of a Weyl fermion semimetal and topological Fermi arcs, Science **349**, 613 (2015).

[12] B. Q. Lv, H. M. Weng, B. B. Fu, X. P. Wang, H. Miao, J. Ma, P. Richard, X. C. Huang, L. X. Zhao, G. F. Chen, Z. Fang, X. Dai, T. Qian, and H. Ding, Experimental Discovery of Weyl Semimetal TaAs, Phys. Rev. X **5**, 031013 (2015).

[13] A. Golub, Shot noise in NS junctions with a Weyl superconductor, Phys. Rev. B **94**, 115133 (2016).

[14] A. A. Burkov, Chiral Anomaly and Diffusive Magnetotransport in Weyl Metals, Phys. Rev. Lett. **113**, 247203 (2014).

[15] D. T. Son and B. Z. Spivak, Chiral anomaly and classical negative magnetoresistance of Weyl metals, Phys. Rev. B **88**, 104412 (2013).

[16] S. A. Parameswaran, T. Grover, D. A. Abanin, D. A. Pesin, and A. Vishwanath, Probing the Chiral Anomaly with Nonlocal Transport in Three Dimensional Topological Semimetals, Phys. Rev. X **4**, 031035 (2014).

[17] G. Sundaram and Q. Niu, Wave-packet dynamics in slowly perturbed crystals: Gradient corrections and Berry-phase effects, Phys. Rev. B **59**, 14915 (1999).

[18] D. Xiao, M.-C. Chang, and Q. Niu, Berry phase effects on electronic properties, Rev. Mod. Phys. **82**, 1959 (2010).

[19] N. Nagaosa *et al.*, Anomalous Hall effect, Rev. Mod. Phys. **82**, 1539 (2010).

[20] Y. Baum, E. Berg, S. A. Parameswaran, and A. Stern, Current at a Distance and Resonant Transparency in Weyl Semimetals, Phys. Rev. X **5**, 041046 (2015).

[21] A. Altland and Dmitry Bagrets, Theory of the strongly disordered Weyl semimetal, Phys. Rev. B **93**, 075113 (2016).

[22] E. Khalaf, M. A. Skvortsov, and P. M. Ostrovsky, Semiclassical electron transport at the edge of a 2D topological insulator: Interplay of protected and unprotected modes, Phys. Rev. B **93**, 125405 (2016).

[23] E. Khalaf and P. M. Ostrovsky, Localization Effects on Magnetotransport of a Disordered Weyl Semimetal, Phys. Rev. Lett. **119**, 106601 (2017).

[24] H. Nielsen and M. Ninomiya, The Adler-Bell-Jackiw anomaly and Weyl fermions in a crystal, Phys. Lett. B **130**, 389 (1983).

[25] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).

[26] A. A. Zyuzin and A. A. Burkov, Topological response in Weyl semimetals and the chiral anomaly, Phys. Rev. B **86**, 115133 (2012).

[27] V. Aji, Adler-Bell-Jackiw anomaly in Weyl semimetals: Application to pyrochlore iridates, Phys. Rev. B **85**, 241101(R) (2012).

[28] Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, 1996).

[29] A. Ya. Shulman and Sh. M. Kogan, Theory of fluctuations in a noneqilibrium electron gas, Sov. Phys. JETP **29**, 467 (1969); S. V. Gantsevich, V. L. Gurevich, and R. Katilius, Fluctuations in semiconductors in a strong electric field and the scattering of light by hot electrons, *ibid.* **30**, 276 (1970).

[30] A. Kamenev, *Field Theory of Non-Equilibrium System* (Cambridge University Press, Cambridge, 2011).

[31] For a review see, e.g., L. S. Levitov, The statistical theory of mesoscopic noise, in *Quantum Noise in Mesoscopic Systems*, edited by Yu. V. Nazarov (Kluwer, Amsterdam, 2003).

[32] D. B. Gutman, A. D. Mirlin, and Y. Gefen, Kinetic theory of fluctuations in conducting systems, Phys. Rev. B **71**, 085118 (2005).

[33] S. Pilgram, Electron-electron scattering effects on the full counting statistics of mesoscopic conductors, Phys. Rev. B **69**, 115315 (2004); S. Pilgram, K. E. Nagaev, and M. Buttiker, Frequency dependent third cumulant of current in diffusive conductors, *ibid.* **70**, 045304 (2004); A. N. Jordan, E. V. Sukhorukov, and S. Pilgram, Fluctuation statistics in networks: A stochastic path integral approach, J. Math. Phys. **45**, 4386 (2004).

[34] T. Bodineau and B. Derrida, Current Fluctuations in Nonequilibrium Diffusive Systems: An Additivity Principle, Phys. Rev. Lett. **92**, 180601 (2004).

[35] K. E. Nagaev, Cascade Boltzmann-Langevin approach to higher-order current correlations in diffusive metal contacts, Phys. Rev. B **66**, 075334 (2002).

[36] K. E. Nagaev, Influence of electron-electron scattering on shot noise in diffusive contacts, Phys. Rev. B **52**, 4740 (1995).

[37] V. I. Kozub and A. M. Rudin, Shot noise in mesoscopic diffusive conductors in the limit of strong electron-electron scattering, Phys. Rev. B **52**, 7853 (1995).