We report the experimental realization of a space-time crystal with tunable periodicity in time and space in a magnon Bose-Einstein condensate (BEC), formed in a room-temperature yttrium iron garnet (YIG) film by a microwave space-homogeneous magnetic field. The magnon BEC is prepared to have a well-defined frequency and nonzero wave vector. We demonstrate how the crystalline “density” as well as the time and space textures of the resulting crystal may be tuned by varying the experimental parameters: External static magnetic field, temperature, thickness of the YIG film, and power of the microwave field. The proposed space-time crystals provide an additional dimension for exploring dynamical phases of matter and can serve as a model nonlinear Floquet system, that brings in touch the rich fields of classical nonlinear waves, magnonics, and periodically driven systems.

DOI: 10.1103/PhysRevB.100.020406

Spontaneous symmetry breaking is a fundamental concept of physics. A well-known example is the breaking of spatial translational symmetry, which leads to a phase transition from fluids to solid crystals. By analogy, one can think about a “time crystal” as the result of breaking translational symmetry in time. More generally, one expects the appearance of a “space-time crystal” as a consequence of breaking translational symmetry both in time and in space. If a time crystal exists it should demonstrate time-periodic motion of its ground state [1]. In addition to the time periodicity, the space-time crystals should be periodic in space, similar to an ordinary crystal.

It was recently argued that time crystals and space-time crystals cannot be realized in thermodynamic equilibrium [2,3]. This led to a search for space-time symmetry breaking phenomena in a wider context, for example, in a system with flux equilibrium [4]. Such oscillatory nonequilibrium states are already well known, starting from string and wind musical instruments, such as violins and organs, through quasiperiodic instabilities in plasma and semiconductors (e.g., the Gunn effect [5] used in radar devices) and photon fields in laser resonators, to a microwave generation by nanosized magnetic resonators, to an injection locking of a microwave oscillator driven by a spin-polarized DC electric current [6].

To narrow down a group of candidates to the space-time crystals, that are close to the original idea of the time crystal [1], a set of requirements was proposed [4,7–10]. These include the following: (i) the level of complexity in the interactions merely sufficient to suppress the dissipation of the injected energy in order to ensure very long energy relaxation time relative to other characteristic times of the interaction processes; (ii) requirement on robustness—indepenence of certain features from the perturbations of the physical system (e.g., the level of disorder); and (iii) spontaneous symmetry breaking, leading to a long-range order and appearance of soft modes of the Nambu-Goldstone type [11].

Possibly the first such example was a periodically driven Floquet quantum system [7], in which the disorder isolates the energy states one from another, suppressing dissipation of the externally pumped energy (see also Refs. [12,13]). Needless to say that none of the previously mentioned examples, such as a violin or the photon field in a laser resonator, meet these criteria.

Natural candidates [14] for time crystals are Bose-Einstein condensates (BECs) in superfluids, trapped cold atoms, and in overpopulated magnon gases [15–19]. Typically, these systems have two characteristic times [15]: The lifetime of the corresponding (quasi)particles \( \tau_N \) and the energy thermalization time \( \tau_\text{th} \) near the bottom of the energy (frequency) spectrum \( E_\text{min} = \hbar \omega_\text{min} \). If \( \tau_\text{th} \ll \tau_N \), the system first quickly thermalizes toward the local thermodynamic equilibrium with an effective chemical potential \( \mu_{\text{eff}} \) and then slowly relaxes further to the global thermodynamic equilibrium. During the time interval \( \tau_\text{th} \ll \tau \ll \tau_N \) and with a sufficiently large number of (quasi)particles, the system reaches the state with \( \mu_{\text{eff}} = E_\text{min} \), creating a BEC, which can be considered as a time crystal with the frequency \( \omega_{\text{min}} \). Perhaps the most recent observation of a time crystal with very long energy relaxation time is the BEC of magnons in a flexible trap in superfluid \(^3\)He-B under periodic driving by an external magnetic field.
The realization of a space-time crystal (STC) with tunable period-
more involved and so far were explored mainly theoretically

5.6-

of the frequency minimum
parallel parametric pumping. Two orange dots indicate the positions
The red arrows illustrate the magnon injection process by means of
[14]. The problems related to space-time crystals [ 20] a r e

The processes leading to the creation of BEC are an experi-
mental manifestation of the STC: A system, driven away
from thermodynamic equilibrium by a space-homogeneous,
time-periodic (with frequency $\omega_p$) pumping field, sponta-
neously chooses a space-time periodic state (1) with the
frequency $\omega_{\text{min}}$ and nonzero wave vectors $\pm q_{\text{min}}$. Importantly,
the parameters $\omega_{\text{min}}$ and $\pm q_{\text{min}}$ are fully determined by in-
trisic interactions in the system and are independent of the
pumping frequency in a wide range of their values. Note that the lifetime $\tau_{\text{th}}$ of the condensate is much longer than
$\tau_{\text{th}}$, enabling the observation of the magnon BEC state and
the study of related effects, such as magnon supercurrents
[18] and Nambu-Goldstone modes—the Bogoliubov waves
[30]. All these meet the presently accepted requirements of
a space-time crystal, listed above.

A more intense pumping process [18,19,31–34] impedes
complete condensation of the scattered magnons by mixing
their frequencies in the near-bottom zone. The resulting two
groups of magnons with a narrow spectral distribution can
be considered as a space-time polycrystal (STPC) with par-
tial coherence. In this case, the interaction-driven magnon
condensation into two coherent spatially extended spin
waves—magnon BECs—is completed after the pumping is
switched off.

Probing the parametrically pumped magnon gas by means of
time-, frequency-, and wave-vector-resolved Brillouin light
scattering (BLS) spectroscopy (see Fig. 2) [35–37], we

FIG. 1. Magnon spectrum of the first 48 thickness modes in a
5.6-μm-thick YIG film magnetized in plane by a bias magnetic field
$H = 1400$ Oe, shown for the wave vector $q \parallel H$ (lower part of
the spectrum, orange curves) and for $q \perp H$ (upper part, blue curves).
The red arrows illustrate the magnon injection process by means of
parallel parametric pumping. Two orange dots indicate the positions
of the frequency minimum $\omega_{\text{min}}(\pm q_{\text{min}})$ occupied by the BECs of
magnons.

In this Rapid Communication, we report the experimental
realization of a space-time crystal (STC) with tunable periodic-
ity in time and space in a magnon BEC formed in a room-
temperature yttrium iron garnet (YIG, Y$_3$Fe$_5$O$_{12}$) film.
The condensate spontaneously arises [17] as a result of scattering
of magnons, parametrically pumped by an intense microwave
field, to the bottom of their spectrum. We show that this
coherent state has the hallmark of nonuniversal relaxation
times, which are much longer than the intrinsic timescales and
the crystallization time. We consider the interacting magnons,
subject to an intensive time-periodic impact, as a model object
for studies of Bose-Einstein condensation process in nonlinear
Floquet wave systems.

The frequency spectrum of magnons in the in-plane mag-
netized ferromagnetic YIG film, shown in Fig. 1, has two
symmetric minima with nonzero frequency and wave vectors
$\omega_{\text{min}} = \omega(\pm q_{\text{min}})$. The possible BEC has, accordingly,
two components with the wave vectors $\pm q_{\text{min}}$ and the frequency
$\omega_{\text{min}}$, creating a standing wave,

$$ C(r,t) = C_0 \cos(q_{\text{min}} \cdot r) \exp(-i\omega_{\text{min}} t). \quad (1) $$

In our experiment (see Fig. 2), we create a BEC by pulsed mi-
crowave radiation of frequency $\omega_p = 2\pi \times 13.6$ GHz, which
can be considered space homogeneous with a wave number
$q_p \approx 0$ [26]. The decay instability of this field with the con-
servation law,

$$ \omega_p \Rightarrow \omega(q) + \omega(-q) = 2\omega(q), \quad \omega(q) = \omega_p/2, \quad (2) $$

excites “parametric” magnons with frequency $\omega(q)$ and wave
vectors $\pm q$ (see Fig. 1). These parametric magnons further
interact mainly via $2 \leftrightarrow 2$ scattering with the conservation
laws,

$$ \omega(q_1) + \omega(q_2) = \omega(q_3) + \omega(q_4), \quad q_1 + q_2 = q_3 + q_4, \quad (3) $$

that preserve the total number of magnons and their energy.
The theory of weak wave turbulence [27,28] shows that the
scattering process (3) leads to a flux of energy towards large
$\omega(q)$ and to a flux of magnons toward small $\omega(q)$, resulting
in an accumulation of magnons near the bottom spectrum
frequency $\omega_{\text{min}}$. The same 2 $\leftrightarrow$ 2 processes lead to an efficient
thermalization of the bottom magnons during some time
$\tau_{\text{th}} \lesssim 50–70$ ns and to the subsequent creation of the BEC
state [17,29].

FIG. 2. Experimental setup. The pumping circuit drives a
50-μm-wide microstrip resonator placed below the YIG film by
microwave pulses of 2 μs duration with a repetition rate of 1 kHz.
Probing light from a solid-state laser ($\lambda = 532$ nm) with a power
of 30 mW is chopped by an acousto-optic modulator (AOM) to
control the energy input into the YIG sample. The frequency-shifted
light, inelastically scattered from magnons, is selected by the tandem
Fabry-Pérot interferometer, detected, and analyzed in time.
register two density peaks of magnons near the bottom of their spectra ($\omega_{\text{min}}, \pm q_{\text{min}}$). The frequency and wave-vector resolutions of our experimental setup (approximately 100 MHz and 2000 cm$^{-1}$, respectively) do not allow for the direct detection of the coherence of the magnon BEC [38]. Therefore, we prove the formation of the magnon BEC in a different way, by observation of its physical consequences. In particular, we observe a magnon supercurrent—a macroscopic flow of the magnon condensate induced by a phase gradient imposed on the BEC’s wave function [18,39]. In the focal spot heated by the probing laser beam (see Fig. 2), the saturation magnetization $M_S$ decreases compared to the rest of the film due to its temperature dependence [40,41], inducing a frequency shift between different parts of the magnon condensate. The resulting phase gradient in the coherent BEC wave function creates a magnon supercurrent, flowing out of the hot spot [18] and leading to a faster decrease of the BLS signal. The detection of such an enhanced decrease allows us to claim the space-time coherence of the observed magnon BEC with a nonzero frequency $\omega_{\text{min}}$ and a nonzero wave number $\pm q_{\text{min}}$ and to identify it as being a space-time crystal.

Now we demonstrate how to control the transition between the STPC and STC magnon phases and how to change all three parameters of the STC described by Eq. (1): The BEC magnon density $|C_0|^2 = N_{\text{BEC}}$, the frequency $\omega_{\text{min}}$, and the wave number $\pm q_{\text{min}}$. Figure 3(a) shows the time evolution of the bottom magnon number $N_b(t)$ in the narrow frequency and wave-number intervals determined by the corresponding setup resolutions. The temperature in the light spot was controlled by the duration of the probing laser pulse $t_L$: The black curve in Fig. 3(a) corresponds to the short duration $t_L = 6 \mu s$, when the light spot can be considered as cold; the colored curves correspond to $t_L = 80 \mu s$, with a hot light spot.

Comparing the black and red curves in Fig. 3(a), measured at the same pumping power of 31 dBm in the cold and hot laser spots, we see a very similar initial evolution of $N_b(t)$ during the action of the pump pulse $t_p = -2000 \text{ ns} < t < 0$. At the first stage of this evolution (first 100 ns after switching on of the pumping), $N_b(t)$ rapidly grows due to magnon flux toward small $\omega(q)$ as a result of $2 \leftrightarrow 2$ scattering. For the rest of the pumping time $-1900 \text{ ns} < t < 0$, the number of bottom magnons remains unchanged both in the cold and in the hot light spots. This means that the heating-induced magnetic space inhomogeneity practically does not affect the evolution of the bottom magnons in this case. The coherent BEC phase is not created during the pump pulse due to a particular type of $2 \leftrightarrow 2$ scattering process (3), in which the gaseous bottom magnons with $\omega(q_3) \approx \omega(q_4) \approx \omega_{\text{min}}$ scatter on the dense group of parametrically pumped magnons with $\omega(q_1) \approx \omega(q_4) \approx \omega_{\text{min}}/2$. This process can be considered as an effective two-magnon ($1 \leftrightarrow 1$) scattering of the bottom magnons leading to their frequency intermixing (about the frequency linewidth of the parametric magnons). As a result, the frequency spread of the bottom magnon density peak $\Delta \omega$ increases up to $2\pi \times 500 \text{ MHz}$. Nevertheless, it is still much smaller than $\omega_{\text{min}}$, or, keeping analogy to the crystals, the autocorrelation time of these incoherent magnons significantly exceeds their wave period, similarly to the autocorrelation length in polycrystals that spans many unit cell sizes. That is why we consider this magnon state as a polycrystalline phase [see the yellow shaded area, marked “Polycrystalline phase-A” in Fig. 3(a) for $t < 0$].

After the pump pulse is switched off ($t > 0$ ns), the parametric magnons quickly disappear, preventing further intermixing of the bottom magnons. For a sufficiently strong pumping power ($P_{\text{pump}} \geq 20 \text{ dBm}$), a part of the bottom magnons creates the BEC, the rest of them remain gaseous. The presence of both BEC and noncoherent magnons is confirmed in Fig. 3(a) by a two-stage decay of the bottom magnon density in the hot light spot (red curve, $P_{\text{pump}} = 31 \text{ dBm}$): A fast decay with the mean lifetime $\langle \tau_{\text{LT}} \rangle \approx 110 \text{ ns}$ for high $N_b(t)$, followed by a much slower decay with $\tau_N \approx 250 \text{ ns}$ for $N_b(t) < N_{\text{LT}} \approx 20 \text{ BLS counts}$. The faster decay is associated with the existence of the magnon BEC for $N_b > N_{\text{LT}}$ [see the blue shaded area in Fig. 3(a) labeled “Space-time crystal”]: The magnon supercurrent flowing out of the hot spot enhances the decay rate in the hot spot region [18]. Later, when $N_b(t) < N_{\text{LT}}$, the magnon BEC and its supercurrent disappear. At this stage, our system returns back to the polycrystalline phase, with an even smaller frequency linewidth $1/\tau_N \approx 2\pi \times 0.7 \text{ MHz}$. This polycrystalline phase-B is shown in Fig. 3(a) by a yellow shaded area for $t > 0$.

It is worth noting that the pumping power of 31 dBm used for the measurement of the black top curve in Fig. 3(a) (cold spot) is the same as for the red curve (hot spot), therefore the black curve also corresponds to a well-formed STC at...
$N_b > N_{cr}$. However, it shows no enhanced decay of the BEC state. The same behavior was observed for all other $P_{\text{pump}}$ (not shown). These observations preclude an interpretation of different relaxation rates of the magnon BEC and the incoherent magnon phase in Ref. [43] as being a result of the sensitivity of the BLS technique to the degree of coherence of the scattering magnons. On the other hand, all observed phenomena are well explained by the emergence of a supercurrent that takes away the coherent magnons from the hot spot.

Important information about the STC-STPC phase transition was obtained by tuning the pumping power $P_{\text{pump}}$ below 31 dBm. Weaker pumping creates a smaller number of parametrically injected magnons, leading to less effective $2 \leftrightarrow 2$ magnon scattering and, thus, to an increasing delay in the appearance of these magnons near the bottom of the energy spectrum [Fig. 3(a), lines marked 19–26 dBm]. The resulting density of the bottom magnons decreases as well. Below the threshold density $N_{cr}$, when the majority of condensed magnons are flown out of the measured region of the BEC, the observed decay rate approaches the same value for all different pumping powers. Importantly, Fig. 3(b) shows that the lifetime of the undisturbed STC in the cold spot is always longer than the lifetime of the corresponding polycrystalline phase. We relate this fact with the continued condensation of gaseous magnons to the BEC phase after the pumping is switched off. These observations clearly prove that the STC density is tunable by the parametric pumping power.

The time periodicity $1/\omega_{\text{min}}$ of both the STC and STPC phases can be easily controlled by variation of the bias magnetic field. For example, in the micrometer-thick YIG films, $\omega_{\text{min}}(H) \approx \gamma H$, where $\gamma$ is the gyromagnetic ratio (see Fig. 4).

The spatial periodicity of the STC can be changed in a wide range from 0.5 to 4 $\mu$m by a proper choice of the YIG film thickness (see Fig. 5). The magnon condensation was already experimentally demonstrated in micrometer-

Financial support by the European Research Council within the Advanced Grant No. 694709 “SuperMagnonics”, by Deutsche Forschungsgemeinschaft (DFG) within the Transregional Collaborative Research Centers SFB/TR49 “Condensed Matter Systems with Variable Many-Body Interaction” (project A7) and SFB/TRR173 “Spin+X – Spin in its collective environment” (project A10) as well as by the DFG Project No. INST 248/178-1 is gratefully acknowledged. D.A.B. acknowledges support from the Alexander von Humboldt Foundation.

[37] O. Büttner, M. Bauer, S. O. Demokritov, B. Hillebrands, Y. S. Kivshar, V. Grimalsky, Y. Rapoport, and A. N. Slavin, Linear and nonlinear diffraction of dipolar spin waves in yttrium iron

[38] In Ref. [14] the coherence of a BEC-related time crystal in $^3$He-$B$ was demonstrated by a direct detection of radio-frequency radiation, caused by the homogeneous precession of the magnetic moment. In YIG films, the typical wave number of the condensed magnons is about $q_{\text{min}} \simeq \pm 10^4$ cm$^{-1}$, four orders of magnitude larger than the wave number of the electromagnetic field with the same frequency. As a result, the direct microwave radiation by condensed magnons is tremendously suppressed and cannot be detected easily. At the same time, the space periodicity of the magnon BEC formed in parametrically driven YIG films has already been evidenced by means of spatially resolved microfocused BLS spectroscopy [32].

[39] Generally speaking, the observation of the supercurrent is not always sufficient to claim the existence of long-distant space coherence. For example, a similar phenomenon was observed in superfluid $^3$He-A, demonstrating a tendency to split into many droplets due to the attractive potential. However, in YIG films under the studied conditions, the interaction potential is repulsive [42] and is preventing such an instability.


