

Lectures:

Monday, 24.6.: Electron-phonon coupling;
 Thursday, 27.6.: Effective Interaction from Schrieffer-Wolff transformation

Exercises:

All solutions must be handed in by **Tue. 2.7.** noon in box on 5th floor of Building 46 or electronically to laschwar@rptu.de

In the lecture we derived a model for electron phonon-coupling. For the case of one phonon-branch and one electron band we write¹

$$H = H_{\text{el}} + H_{\text{ph}} + H_{\text{el-ph}} = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger \psi_{\mathbf{k}} + \hbar \omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{q}} T(\mathbf{q}) \psi_{\mathbf{k}}^\dagger \psi_{\mathbf{k}+\mathbf{q}} (a_{\mathbf{q}}^\dagger + a_{-\mathbf{q}}) \right),$$

where $T(\mathbf{q}) = T^*(-\mathbf{q})$ is a small coupling parameter. In order to decouple the phonon excitations, we suggested a Schrieffer-Wolff transformation of the form

$$S = \sum_{\mathbf{k}_1, \mathbf{q}_1} T(\mathbf{q}_1) \psi_{\mathbf{k}_1}^\dagger \psi_{\mathbf{k}_1 + \mathbf{q}_1} (x a_{\mathbf{q}_1}^\dagger - y a_{-\mathbf{q}_1})$$

17a.) Generally all electron operators Λ commute with all phonon operators Ω . Show that for general products $\Lambda\Omega$ and $\Lambda'\Omega'$, the following identity holds:

$$[\Lambda\Omega, \Lambda'\Omega'] = [\Lambda, \Lambda']\Omega\Omega' + [\Omega, \Omega']\Lambda'\Lambda$$

b) Provide the missing steps from the lecture on slide 10-11 in order to calculate the commutator $H_{\text{el-ph}} = [S, H_{\text{el}} + H_{\text{ph}}]$, from which the parameters x and y are determined.

¹ The summation over electron spin quantum numbers is implied (can be ignored for this exercise).