Lectures:

Monday, 3.6.: Interactions in the Hydrogen molecule, Hubbard model
Thursday, 6.6.: Mott insulators, Fermi liquids

Exercises:

All solutions must be handed in by **Tue. 11.6.** noon in box on 5th floor of Building 46 or electronically to laschwar@rptu.de

We consider the two-site Hubbard Model as discussed in the lecture for the Hydrogen molecule $H = -t \sum_{\sigma=\uparrow,\downarrow} \left(\psi^{\dagger}_{A,\sigma} \psi_{B,\sigma} + \psi^{\dagger}_{B,\sigma} \psi_{A,\sigma} \right) + U \left(n_{A,\uparrow} n_{A,\downarrow} + n_{B,\uparrow} n_{B,\downarrow} \right)$

12.) Express the Hamiltonian in the even/odd basis $\psi^{\dagger}_{0/\pi,\sigma} = \frac{1}{\sqrt{2}} \left(\psi^{\dagger}_{A,\sigma} \pm \psi^{\dagger}_{B,\sigma} \right)$ to show that the hopping becomes diagonal, while the interaction becomes non-diagonal, i.e. involves scattering. Show that the spin-polarized two-particle state $\psi^{\dagger}_{A,\uparrow} \psi^{\dagger}_{B,\uparrow} |0\rangle$ is an eigenstate of the Hamiltonian and determine the energy.

13.) List the six[1] two-particle basis states $\psi^{\dagger}_{i,\sigma'} \psi^{\dagger}_{j,\sigma} |0\rangle$. The Hamiltonian is invariant under parity $A \leftrightarrow B$ and conserves spin and particle number. Using the fact that operators anti-commute, identify two possible linear combination of two-particle basis states, which are symmetric under parity $A \leftrightarrow B$ and in a singlet state, i.e. anti-symmetric under spin reversal. (This corresponds to the symmetry properties of the Heitler London ansatz). Determine the 2x2 Matrix for the Hamiltonian in this sub-space and calculate the eigenenergies.

[1] There are $\binom{4}{2} = 6$ possibilities to distribute 2 electrons on 4 single particle states.