

**Lectures:**

Monday, 29.4.: Review. Phonon specific heat, structure factor.

Thursday, 2.5.: Neutron scattering, anharmonic effects.

Monday, 6.5.: Electron wavefunction and “second quantization”.

**Exercises:** to be handed in by **Tue. 14.5.** noon in box on 5<sup>th</sup> floor of Building 46 or electronically to laschwar@rptu.de

4) In the lecture it was claimed that for thermal expectation values of an arbitrary linear combination of bosonic operators  $F = \sum_{\alpha, \vec{k}} (f_{\alpha, \vec{k}} a_{\alpha, \vec{k}} + g_{\alpha, \vec{k}} a_{\alpha, \vec{k}}^\dagger)$  the following equality

$$\text{holds: } \langle e^F \rangle = e^{\langle F^2 / 2 \rangle}.$$

- a) Show this equality for a single oscillator mode  $F = f a + g a^\dagger$  for  $T=0$  (ground state expectation).
- b) Evaluate  $e^{\langle F^2 / 2 \rangle}$  in terms of  $f$  and  $g$  for the finite temperature case.
- c) Argue that  $\langle e^F \rangle = e^{\langle F^2 / 2 \rangle}$  is valid for any number of phonon modes if they are independent and express  $\langle F^2 / 2 \rangle$  as a sum over the 1BZ.

5.) Consider the Debye-Waller factor  $e^{-2W}$  with  $2W = \langle (\bar{q} \cdot \vec{X}_j)^2 \rangle$ .

- a) Evaluate  $2W = \langle (\bar{q} \cdot \vec{X}_j)^2 \rangle$  explicitly by inserting the expression of  $\vec{X}_j$  in terms of normal phonon modes (see lecture 2-10). The final results should correspond to a sum over the 1BZ.
- b) The Debye approximation replaces the sum over the 1BZ by an integral over the sphere  $k < k_D$  and assumes a constant velocity independent of direction  $\alpha = x, y, z$ . Use this approximation to simplify the expression for  $2W = \langle (\bar{q} \cdot \vec{X}_j)^2 \rangle$ . Find an analytic expression for the integral for the low and the high temperature behavior as a function of  $T/T_D$ .