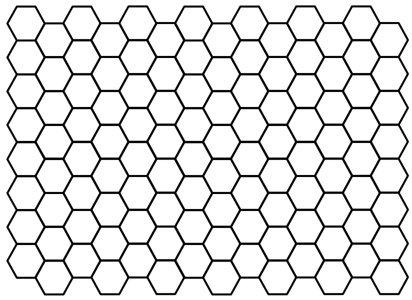
Lectures:

Monday, 22.4.: Introduction, Bravais lattice, reciprocal lattice, Fourier transformations.
Thursday, 25.4.: Lattice vibrations. Dynamical matrix. Phonon eigenmodes, Dispersion. Creation and annihilation operators.

Exercises: must be handed in by **Tue. 7.5.** noon in box on 5th floor of Building 46 or electronically to laschwar@rptu.de

1) Carbon is naturally found in form of graphite: stacks of honeycomb lattices, sp^2 hybridized, distance c=1.421Å at an angle of 120°. Single layers of graphene can be isolated using the "scotch tape technique" (Nobel Prize 2010).



a) What are suitable basis vectors? How many atoms are in each unit cell? Determine the Wigner-Seitz unit cell. Is it possible to make the unit cell rectangular? What is the lattice constant? Give all vectors in units of c and draw them together with a unit cell.

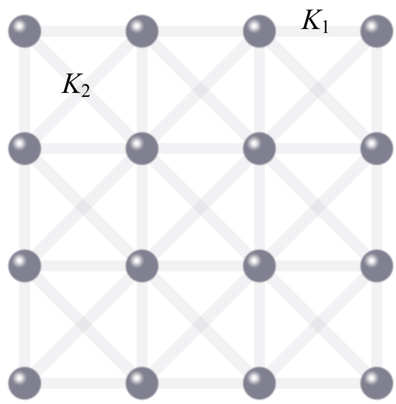
b) Determine the primitive vectors of the reciprocal lattice and draw them together with the 1st Brillouin zone (1BZ).

2) Consider a periodic chain of N unit cells each with two atoms of masses of m_1 and m_2, which are coupled by harmonic "springs" of strength K

$$H = \sum_{n=1}^{N}\left(\frac{p_1^2(n)}{2m_1} + \frac{p_2^2(n)}{2m_2}\right) + \frac{K}{2}\sum_n \left((u_1(n) - u_2(n))^2 + (u_2(n) - u_1(n+1))^2 \right)$$

Use a Fourier transformation to calculate the dynamical Matrix $D^{12}(k)$. Determine the dispersion relations of the corresponding two eigenmodes and sketch the solution for m_1=2m_2.

3) Consider atoms with mass m that are confined in a plane on a square lattice with lattice constant a and horizontal springs K_1 and diagonal springs K_2. In the spring model the coupling is proportional to the quadratic displacement along the fixed direction of each spring (i.e. only the lengths of the springs are effectively changed by the displacement, not the direction, which would be a higher order effect).



a) Write the potential energy as a sum over products of displacements at each lattice points and determine the corresponding dynamical matrix in the 1BZ using a two-dimensional Fourier transformation.

b) Assume small wavevectors $|\boldsymbol{k}|$ to find the velocities of sound for the two eigenmodes along the horizontal and diagonal directions in the 1BZ.