Off Diagonal Long Range Order in the Bose Hubbard Model

$$\hat{H}_{BH} = -t\sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + U\sum_j \hat{n}_j(\hat{n}_j - 1) - \mu \sum_j \hat{n}_j$$

$$G_1(i-j) = \left\langle \hat{a}_i^\dagger \hat{a}_j \right\rangle$$

Mean Field Ansatz

$$\hat{a}_i^\dagger \hat{a}_j = \hat{a}_i^\dagger \left\langle \hat{a}_j \right\rangle + \left\langle \hat{a}_i^\dagger \right\rangle \hat{a}_j - \left\langle \hat{a}_i^\dagger \right\rangle \left\langle \hat{a}_j \right\rangle$$

$$\left\langle \hat{a}_j \right\rangle = \psi$$

$$\hat{H}_{BH} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + U \sum_j \hat{n}_j (\hat{n}_j - 1) - \mu \sum_j \hat{n}_j$$

Mean Field Calculation

$$\hat{H}_{MF} = -t\sum_{\langle i,j\rangle}\left(\hat{a}_i^{\dagger}\psi + \psi^* \hat{a}_j - |\psi|^2\right) + U\sum_j \hat{n}_j(\hat{n}_j - 1) - \mu\sum_j \hat{n}_j$$

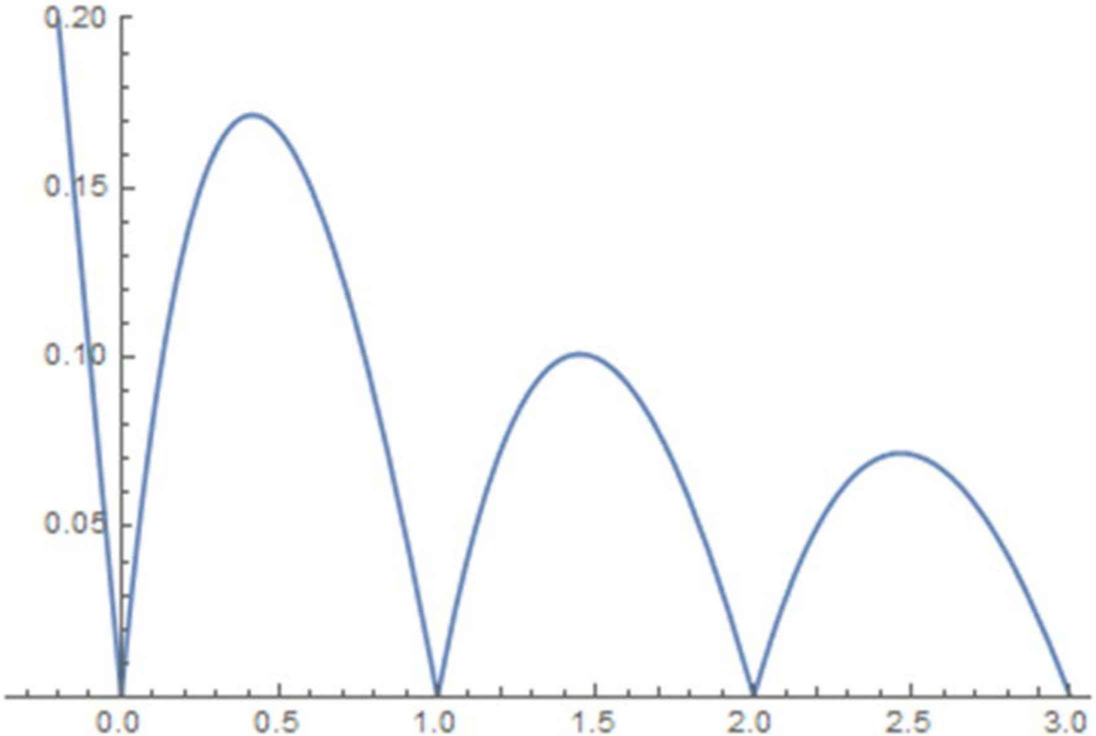
Condition at phase transition: $\left\langle \hat{a}_j \right\rangle = \psi \to 0$

$$\hat{H}_0 = U\hat{n}(\hat{n}-1) - \mu\hat{n} \qquad\qquad \hat{H}_1 = -tz\left(\hat{a}^\dagger\psi + \psi^*\hat{a} - |\psi|^2\right)$$

Phase transition line

Boson localization and the superfluid-insulator transition
Matthew P. A. Fisher, Peter B. Weichman, G. Grinstein, and Daniel S. Fisher
Phys. Rev. B 40, 546 – Published 1 July 1989

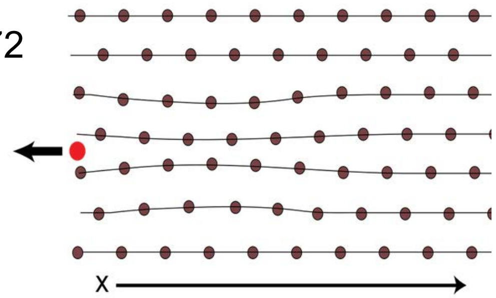
$$zt = \frac{(\mu - nU)((n-1)U - \mu)}{U + \mu}$$

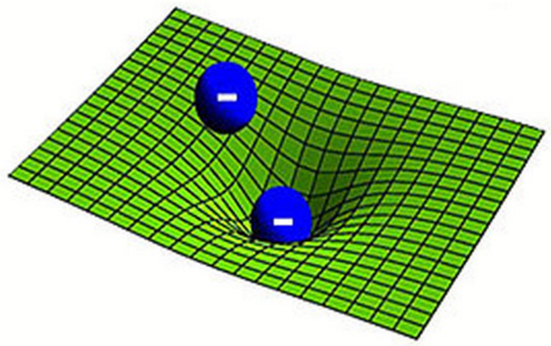


Superconductivity

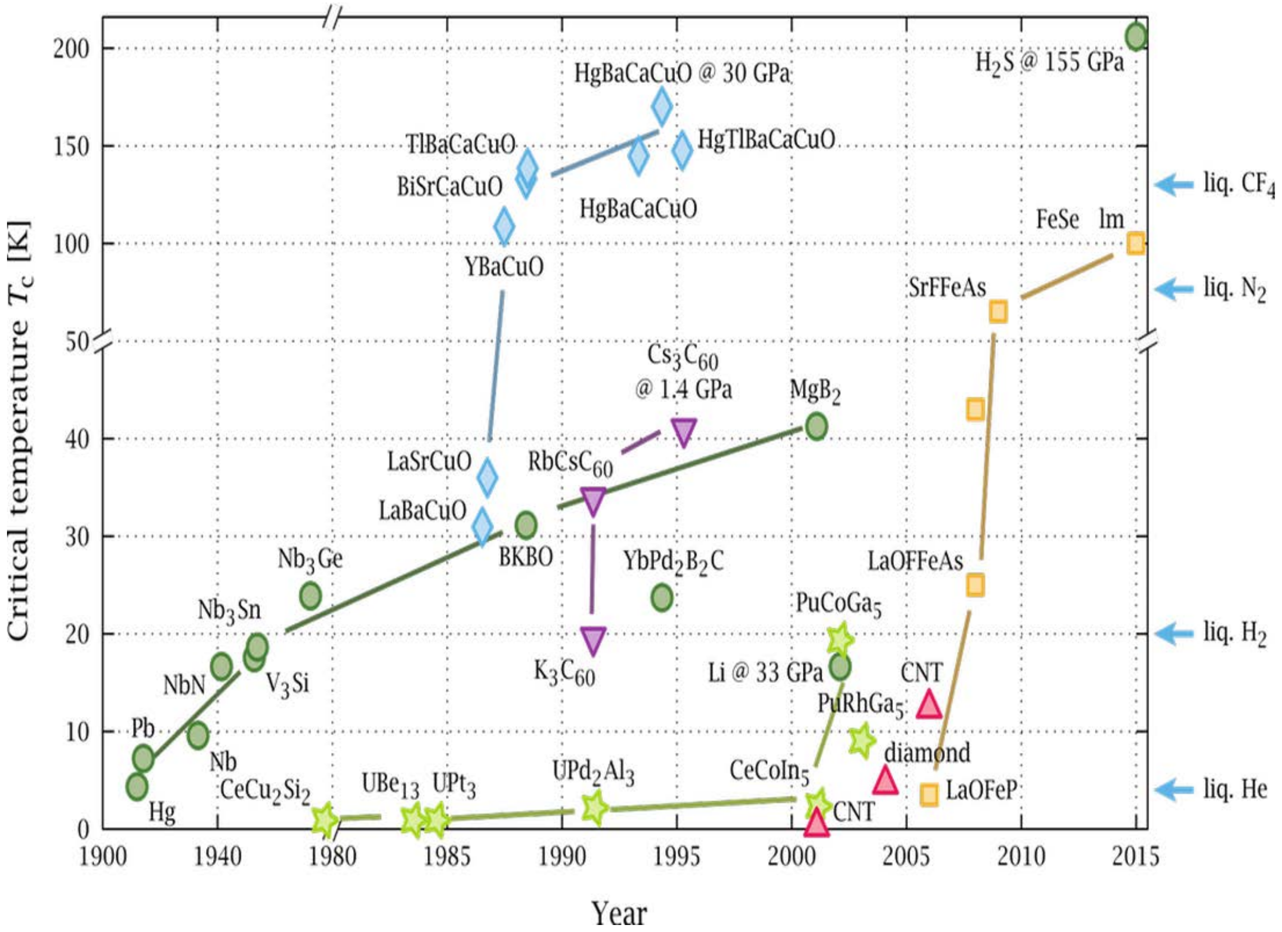
History

1911 Onnes (and Holst) discover superconductivity in mercury	Nobel prize 1913
1933 Meissner Effect	
1935 London equations.	
1935 Shubnikov: Type-II Superconductivity	
1950 Ginzburg-Landau equations	Nobel prize 1962 and 2003
1950 Isotope effect	
1952 Fröhlich Hamiltonian	
1955 Bardeen predicts gap	Nobel prize 1956 and 1972
1956 Gap confirmed	
1956 Cooper pairing	
1957 Schrieffer wave function. BCS Theory	Nobel prize 1972
1962 Josephson effect	Nobel prize 1973
1986 High temperature superconductivity	Nobel prize 1987

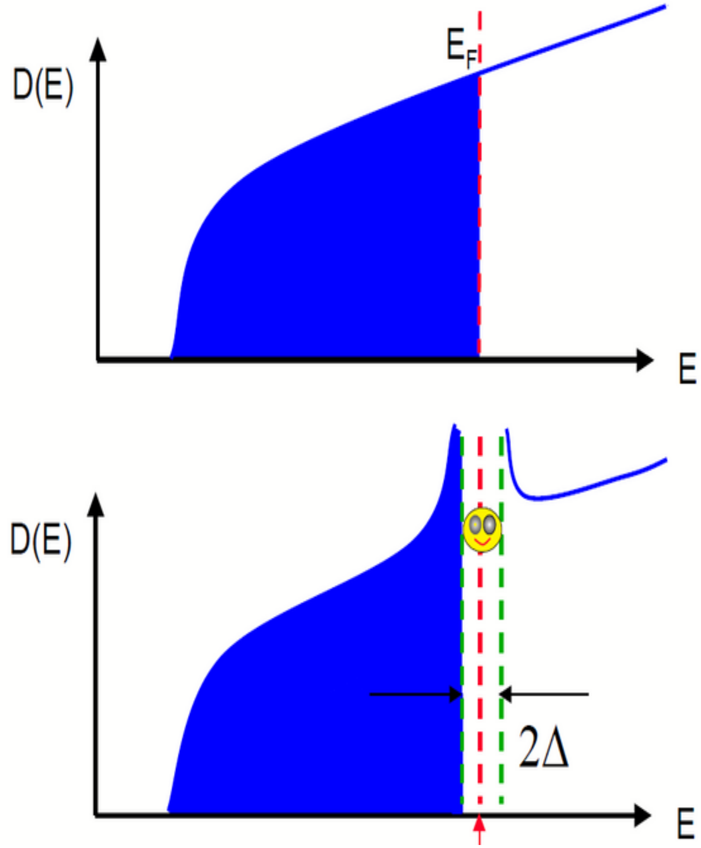




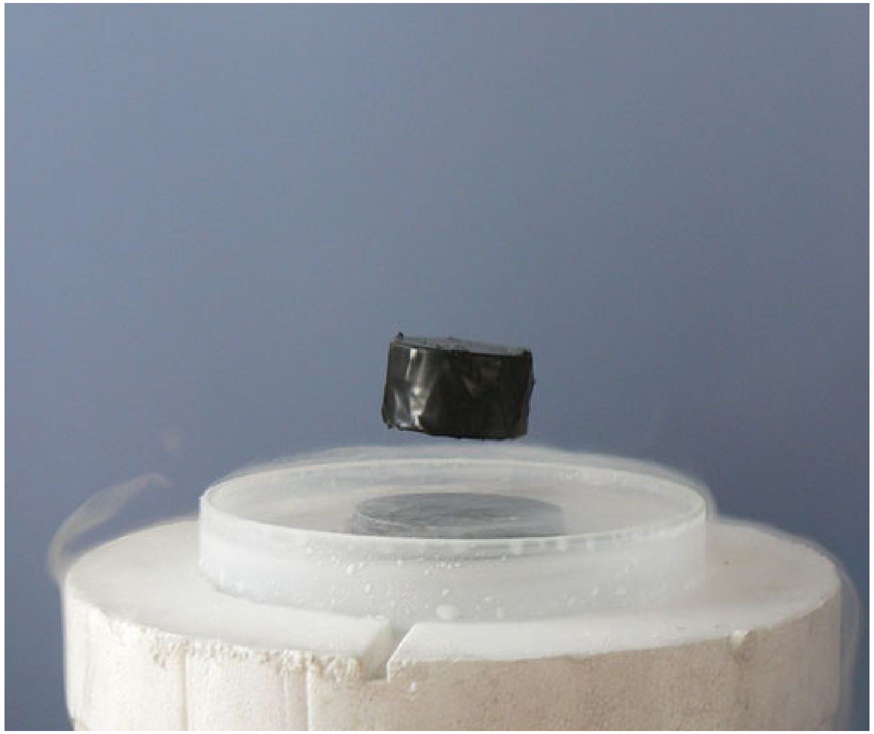
Superconducting Materials

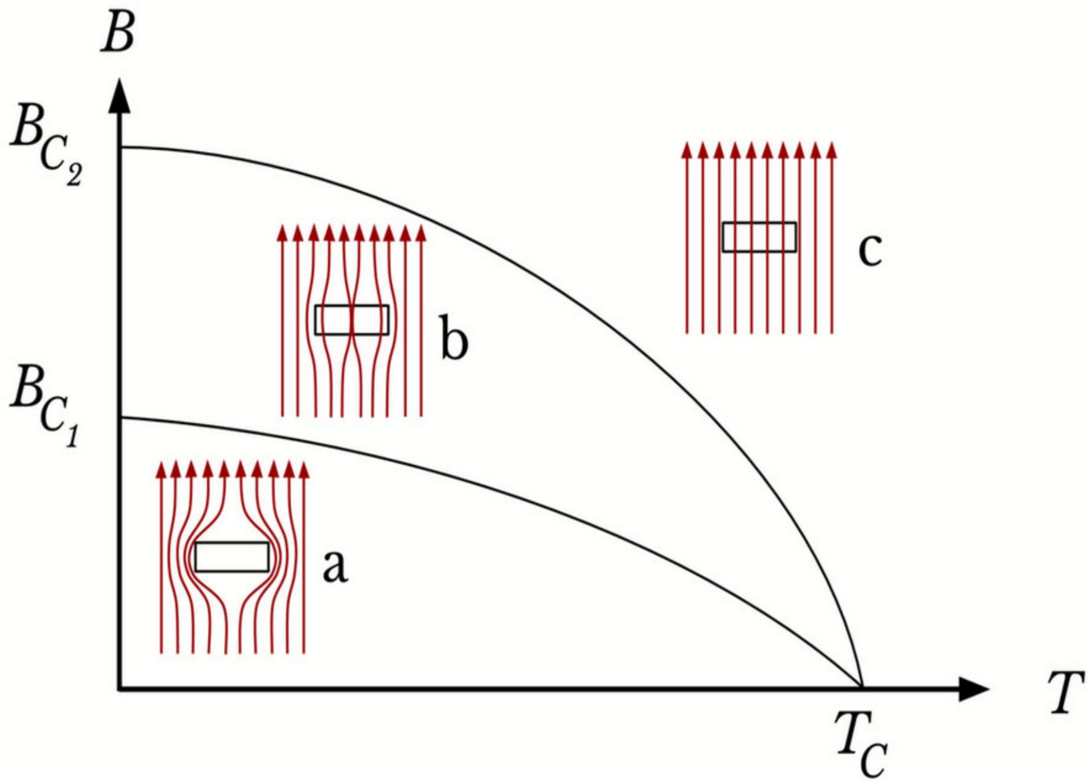


superconductor gap



Meisner effect and type-I and type-II superconductivity





Phenomenological approach: London equations (1935)

Background: Maxwell equations

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

Assumption: Some electrons can move without scattering

For superconducting density n_s

$$\frac{\partial \vec{j}}{\partial t} = \frac{n_s e^2}{m}\vec{E}$$

Vector potential

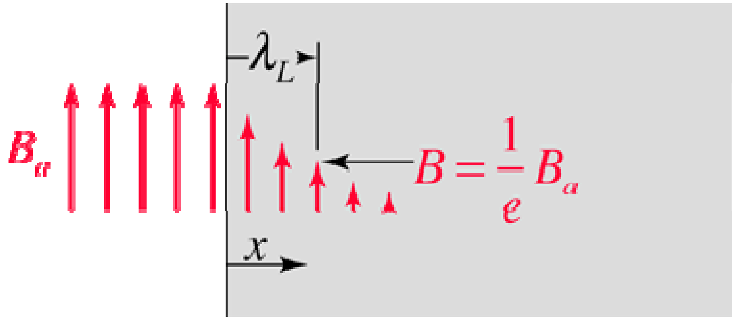
$$\vec{E} = -\vec{\nabla}\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{j} = -\frac{n_s e^2}{mc}\vec{B}$$

Steady state solution of London equations

$$\frac{n_s e^2}{m}\vec{E} = \frac{\partial \vec{j}}{\partial t}$$



$$-\frac{n_s e^2}{mc}\vec{B} = \vec{\nabla} \times \vec{j}$$

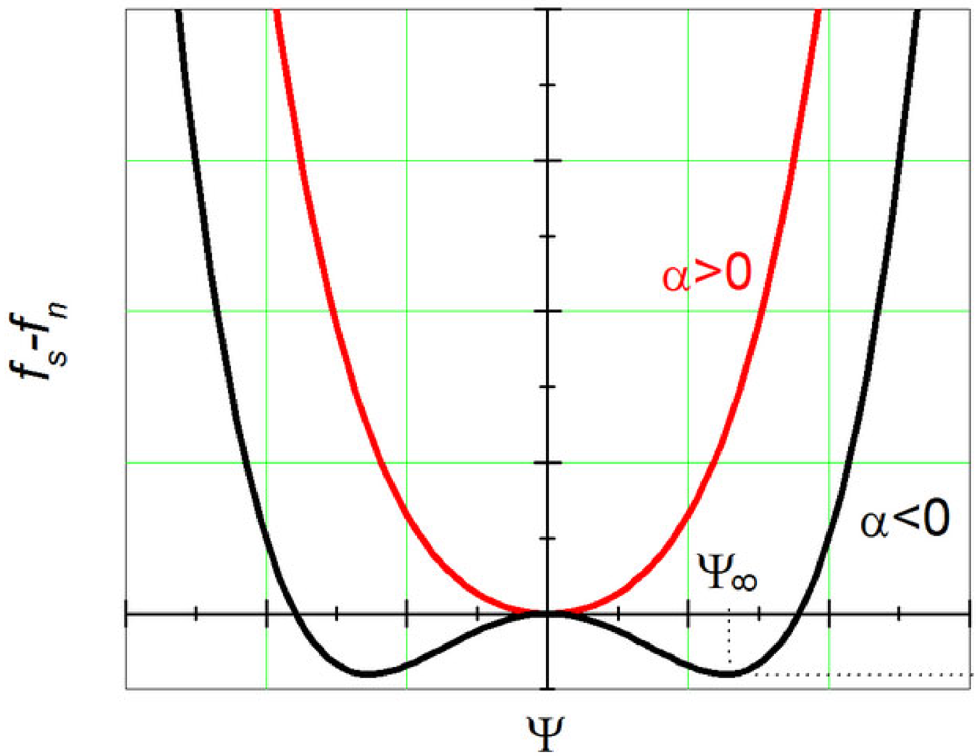
Materia l	Coherence length ξ_0(nm)	London penetration depth λ_L(nm)	Ratio λ_L/ξ_0
Sn	230	34	0.16
Al	1600	16	0.010
Pb	83	37	0.45
Cd	760	110	0.14
Nb	38	39	1.02

Current as an expectation value of a wave-function ψ

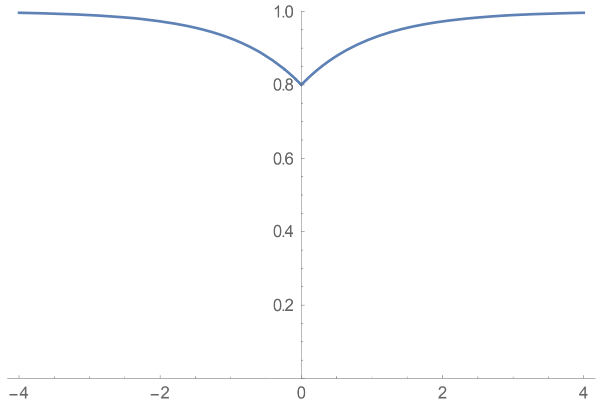
$$\vec{j} = \frac{e}{2m}\left(\psi^* \vec{P}\psi + \psi(\vec{P}\psi)^*\right) \qquad \vec{P} = -i\hbar\vec{\nabla} - \frac{e}{c}\vec{A}$$

Ginzburg Landau effective free energy density for ψ

$$f = \frac{\left|\vec{P}\psi\right|^2}{2m} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$



The healing length: differential equation for ψ



$$\frac{\partial}{\partial \psi^*} f = 0 = \frac{\vec{P}^2}{m}\psi + 2\alpha\psi + 2\beta|\psi|^2\psi$$