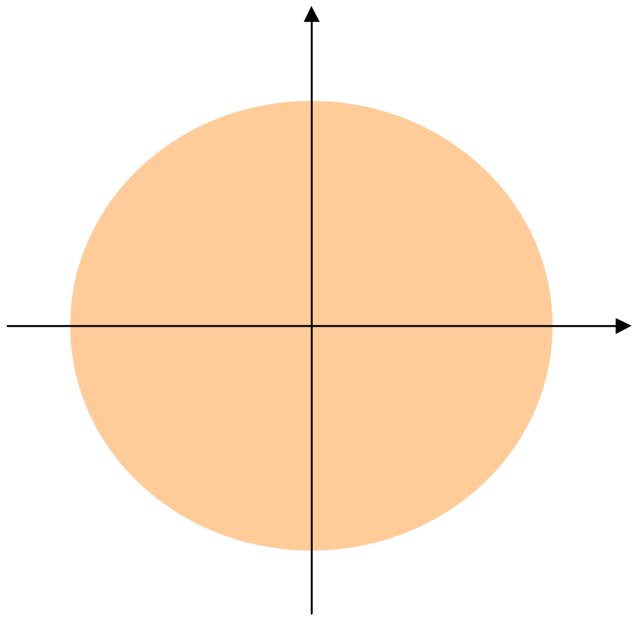
## Stability of the excitations

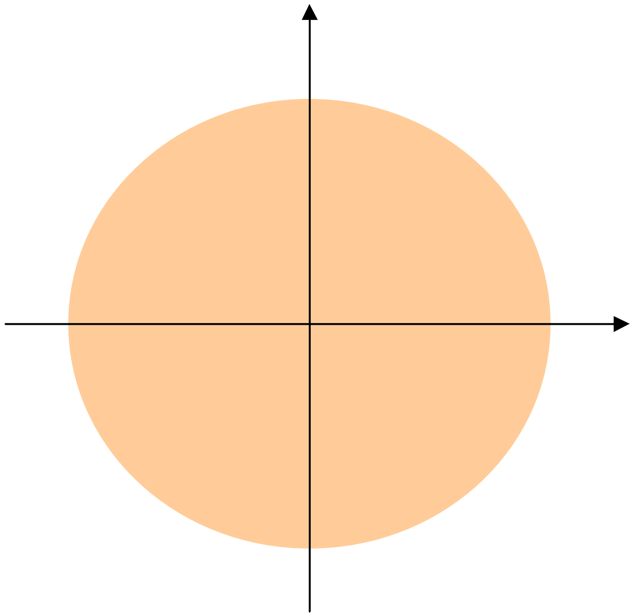
Decay probability using two constraints

Inverse lifetime is proportional



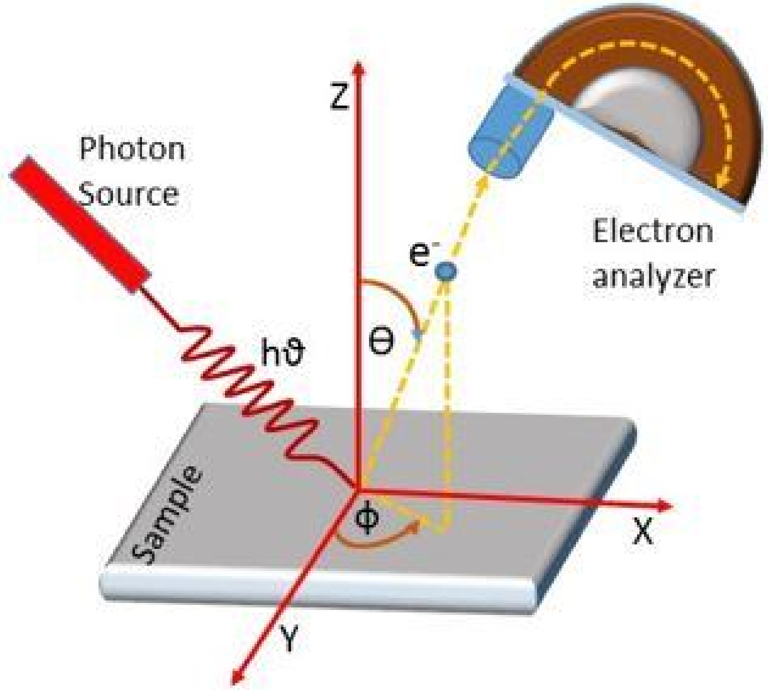
## Effective lifetime and correlations

$$c_\sigma^\dagger(\vec{k},t) = e^{-it\varepsilon_{\vec{k}}/\hbar} c_\sigma^\dagger(\vec{k})$$



## Spectral density: ARPES

$$\rho(\omega,\vec{k}) = \sum_{\lambda} \left| \langle \lambda | c_\sigma(\vec{k}) | GS \rangle \right|^2 \delta(\omega - \omega_\lambda)$$



## Retarded Green's Function

$$\rho(\omega,\vec{k}) = -\frac{1}{\pi}\operatorname{Im} G_R(\omega,\vec{k})$$

where $$G_R(\omega,\vec{k}) = -i\int_0^\infty dt e^{i\omega t}\left\langle\left\{c_\sigma(\vec{k},t), c_\sigma^\dagger(\vec{k},0)\right\}\right\rangle$$

## Self Energy and quasi-particle spectral weight

$$c_\sigma(\vec{k},t) = e^{i\omega_{\vec{k}}t} e^{-i\Gamma_{\vec{k}}t} c_\sigma(\vec{k})$$

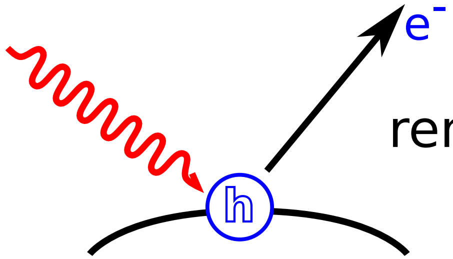
$$\rho(\omega,\vec{k}) = -\frac{1}{\pi}\operatorname{Im} G_R(\omega,\vec{k})$$

$$G_R(\omega,\vec{k}) = \frac{Z}{\omega - \omega_{\vec{k}} - \Sigma_{\vec{k}}}$$

# Spectral functions

**Spectral function**
structures at poles of *G*

$$\rho(\omega) = \frac{1}{\pi}\left|\mathrm{Im}G(\omega)\right|$$



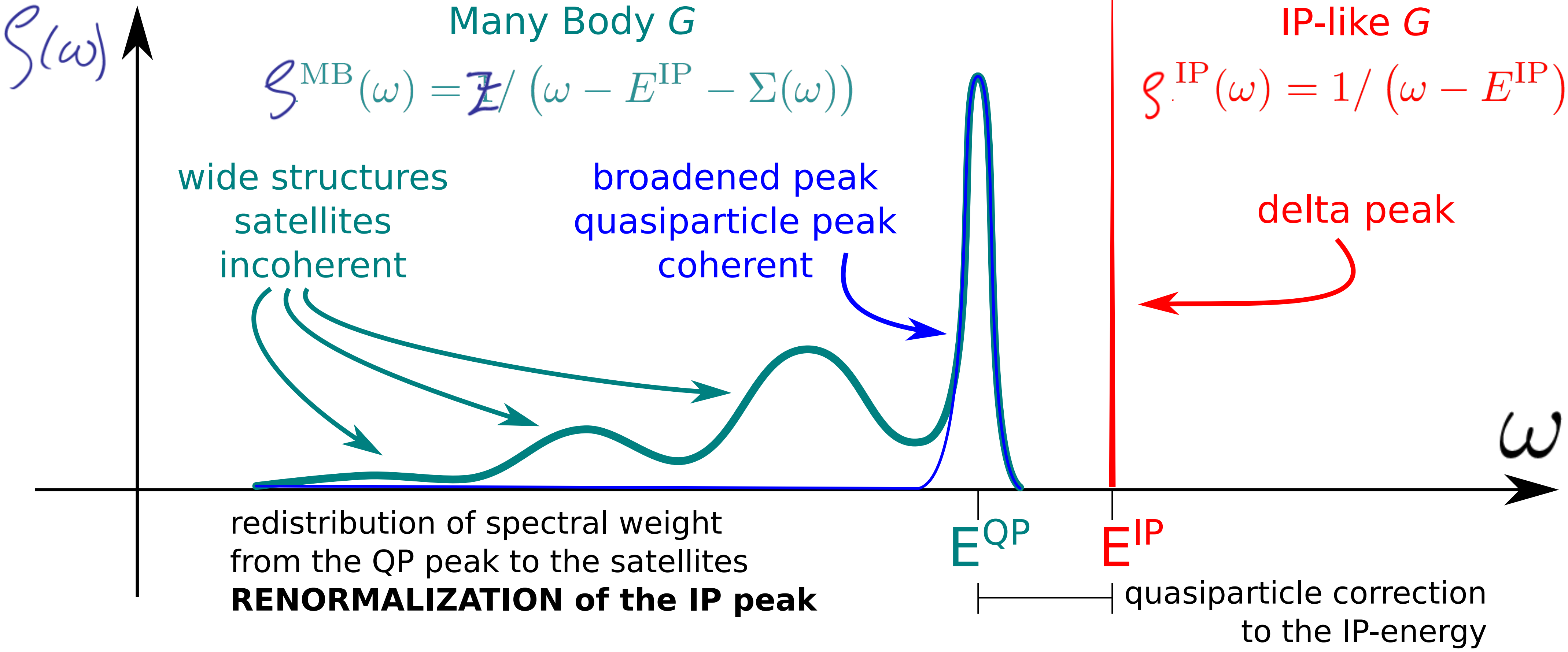

**Photoemission**
removal energy (hole creation)

**Dynamic self-energy**
Many Body *G*

$$\rho^{\mathrm{MB}}(\omega) = Z/\left(\omega - E^{\mathrm{IP}} - \Sigma(\omega)\right)$$

**Static Hamiltonian**
IP-like *G*

$$\rho^{\mathrm{IP}}(\omega) = 1/\left(\omega - E^{\mathrm{IP}}\right)$$




redistribution of spectral weight
from the QP peak to the satellites
**RENORMALIZATION of the IP peak**

## General Mean Field Theory

e.g. $\hat{H}_{\text{int}} = \sum_{\sigma_1,\sigma_2} \int d^3\vec{r}_1 d^3\vec{r}_2 \; \hat{\Psi}^\dagger_{\sigma_1}(\vec{r}_1)\hat{\Psi}^\dagger_{\sigma_2}(\vec{r}_2)V(\vec{r}_1-\vec{r}_2)\hat{\Psi}_{\sigma_2}(\vec{r}_2)\hat{\Psi}_{\sigma_1}(\vec{r}_1)$

or Hubbard interaction $Un_{j,\uparrow}n_{j,\downarrow}$

$$\hat{H}_{\text{int}} = \hat{A}\hat{B}$$

## Hartree Fock Theory

Variational ansatz for a product state with $N$ particles $\left|\psi_{HF}\right\rangle = \Psi_1^\dagger \Psi_2^\dagger \Psi_3^\dagger \ldots \Psi_N^\dagger \left|0\right\rangle$

where an arbitrary single particle wavefunction is described by $\Psi_1^\dagger \left|0\right\rangle = \int d^3\vec{r}\; \varphi_1(\vec{r}) \hat{\Psi}^\dagger(\vec{r}) \left|0\right\rangle$

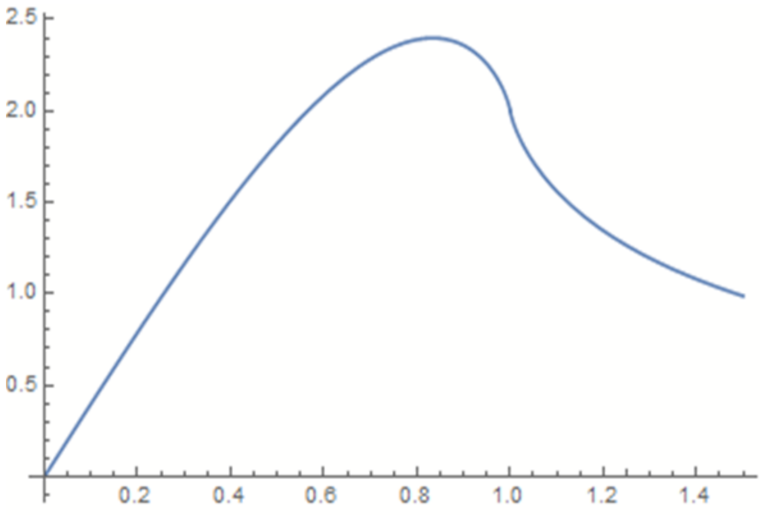
General two-body interaction $\widehat{H}_{\text{int}} = \sum \hat{\Psi}_a^\dagger \hat{\Psi}_b^\dagger V \hat{\Psi}_c \hat{\Psi}_d$

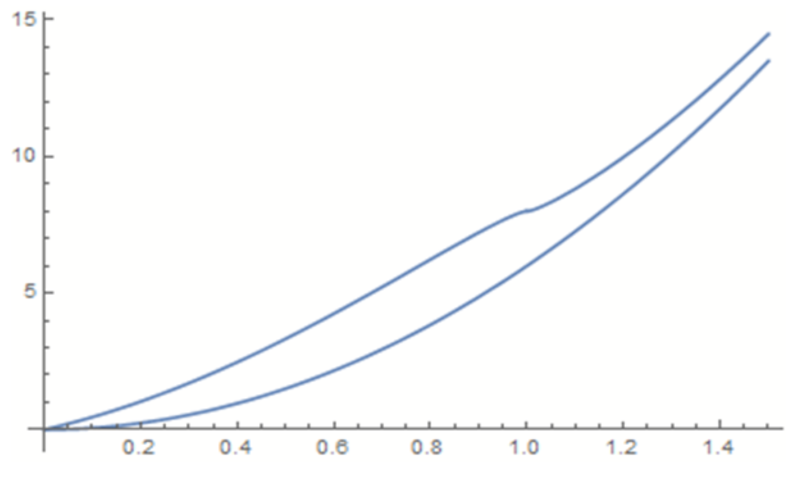
## Hartree and Fock terms as a mean fields:

$$c^{\dagger}_{\sigma_1}(\vec{k}_1)c^{\dagger}_{\sigma_2}(\vec{k}_2)\widetilde{V}(\Delta\vec{k})c_{\sigma_2}(\vec{k}_2-\Delta\vec{k})c_{\sigma_1}(\vec{k}_1+\Delta\vec{k})$$

## Example: Coulomb gas

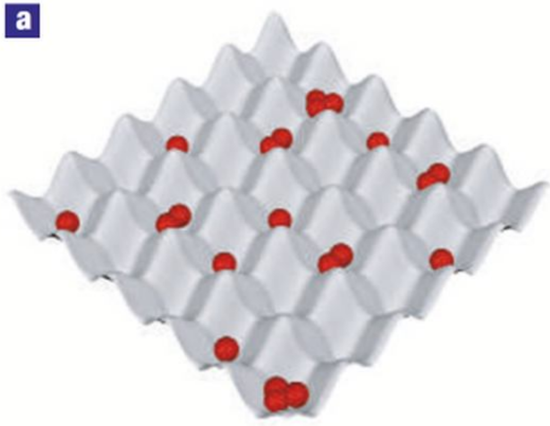
$$\tilde{V}(\Delta\vec{k}) = \int d^3\vec{r}\, e^{i\vec{r}\cdot\Delta\vec{k}} \frac{e^2}{(2\pi)^3 r} = \frac{e^2}{2\pi^2 |\Delta\vec{k}|^2}$$

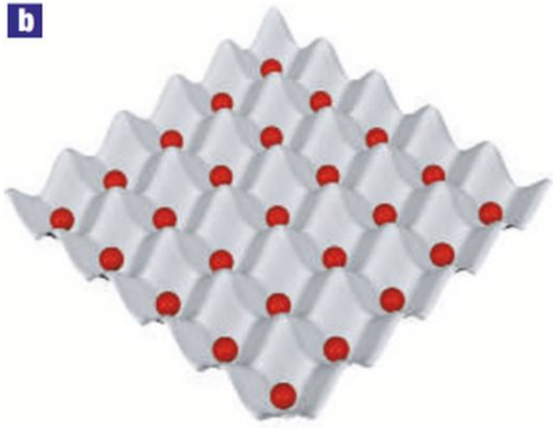


## The Bose Hubbard model

$$\hat{H}_{BH} = -t\sum_{\langle i,j\rangle}\hat{a}_i^\dagger\hat{a}_j + U\sum_j \hat{n}_j(\hat{n}_j - 1) - \mu\sum_j \hat{n}_j$$



## Off Diagonal Long Range Order

$$G_1(i-j)=\left\langle \hat{a}_i^{\dagger}\hat{a}_j\right\rangle$$

## Mean Field Ansatz

$$\hat{a}_i^\dagger \hat{a}_j = \hat{a}_i^\dagger \left\langle \hat{a}_j \right\rangle + \left\langle \hat{a}_i^\dagger \right\rangle \hat{a}_j - \left\langle \hat{a}_i^\dagger \right\rangle \left\langle \hat{a}_j \right\rangle$$

$$\left\langle \hat{a}_j \right\rangle = \psi$$

$$\hat{H}_{BH} = -t \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + U \sum_j \hat{n}_j (\hat{n}_j - 1) - \mu \sum_j \hat{n}_j$$

## Mean Field Calculation

$$\hat{H}_{MF} = -t\sum_{\langle i,j\rangle}\left(\hat{a}_i^{\dagger}\psi + \psi^{*}\hat{a}_j - |\psi|^2\right) + U\sum_j \hat{n}_j(\hat{n}_j - 1) - \mu\sum_j \hat{n}_j$$

**Condition at phase transition:** $\langle \hat{a}_j \rangle = \psi \rightarrow 0$

$$\hat{H}_0 = U\hat{n}(\hat{n}-1) - \mu\hat{n}$$

$$\hat{H}_1 = -tz\left(\hat{a}^\dagger \psi + \psi^* \hat{a} - |\psi|^2\right)$$

## Phase transition line

$$zt = \frac{(\mu - nU)((n-1)U - \mu)}{U + \mu}$$

Boson localization and the superfluid-insulator transition
Matthew P. A. Fisher, Peter B. Weichman, G. Grinstein, and Daniel S. Fisher
Phys. Rev. B 40, 546 – Published 1 July 1989