Coupled Spins

Hund's rules

Exchange and Superexchange

Dipolar Couplings

Heisenberg Model

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

XY-Model

$$H = \sum_{\langle i,j \rangle} J_{ij} (S_i^x \cdot S_j^x + S_i^y \cdot S_j^y)$$

Ising-Model

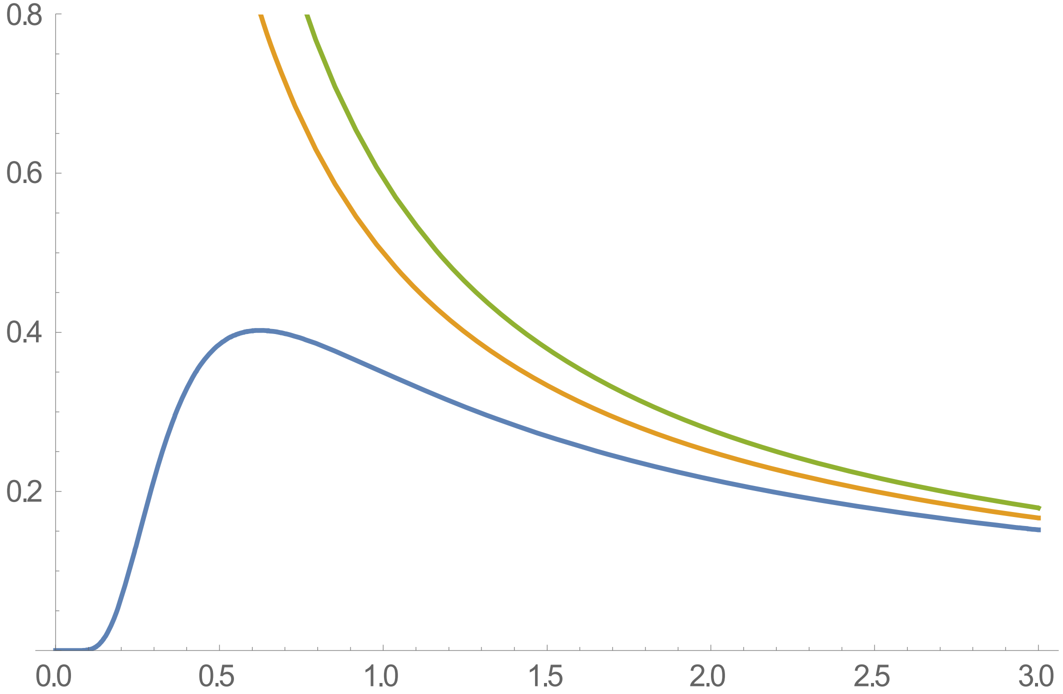
$$H = \sum_{\langle i,j \rangle} J_{ij} S_i^z \cdot S_j^z$$

Example: Two coupled Spins in a Field

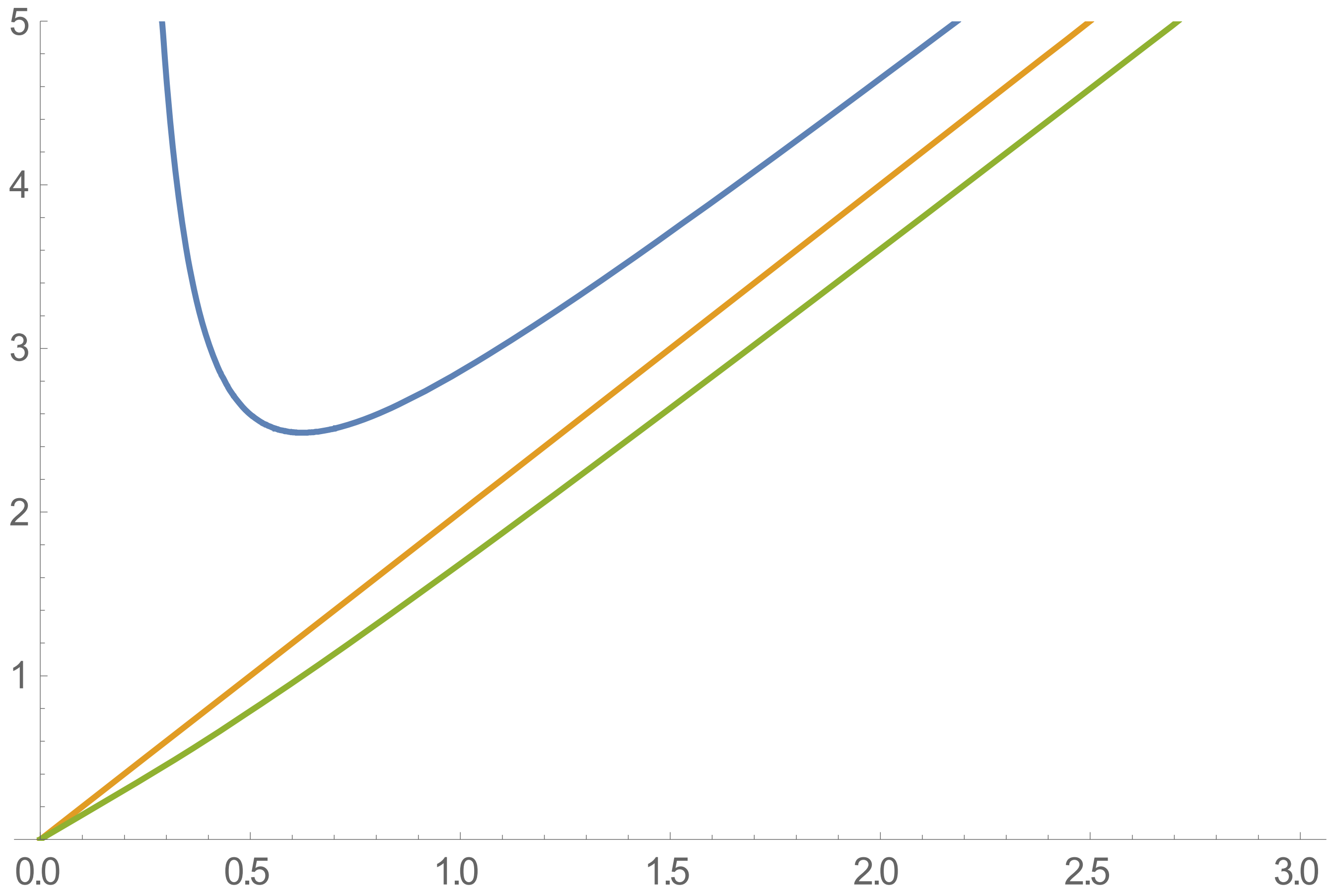
$$H = J\vec{S}_1 \cdot \vec{S}_2 - B(S_1^z + S_2^z)$$

Dimer Susceptibility

$$\chi = \frac{g^2 \mu_B^2}{k_B T} \left\langle (S_{tot}^z)^2 \right\rangle$$



Ferromagnetic, Paramagnetic and Antiferromagnetic Susceptibilities



High temperature expansion

$$\chi = \frac{g^2 \mu_B^2}{k_B T} \left\langle (S_{tot}^z)^2 \right\rangle = \frac{g^2 \mu_B^2}{k_B T} \left(\sum_i \left\langle (S_i^z)^2 \right\rangle + \sum_{i \neq j} \left\langle S_i^z S_j^z \right\rangle \right)$$

$$\left\langle S_i^z S_j^z \right\rangle = \frac{\sum_{\{S_i^z\}} S_i^z S_j^z e^{-\beta H}}{Z}$$

Example: Heisenberg Model

$$H = \sum_{n,m} J_{nm} \vec{S}_n \cdot \vec{S}_m - g\mu_B \sum_n B S_n^z$$

$$\left\langle S_i^z S_j^z \right\rangle = \frac{\sum_{\{S_i^z\}} S_i^z S_j^z e^{-\beta H}}{Z}$$

Curie-Weiss Susceptibility and Curie-Weiss Temperature

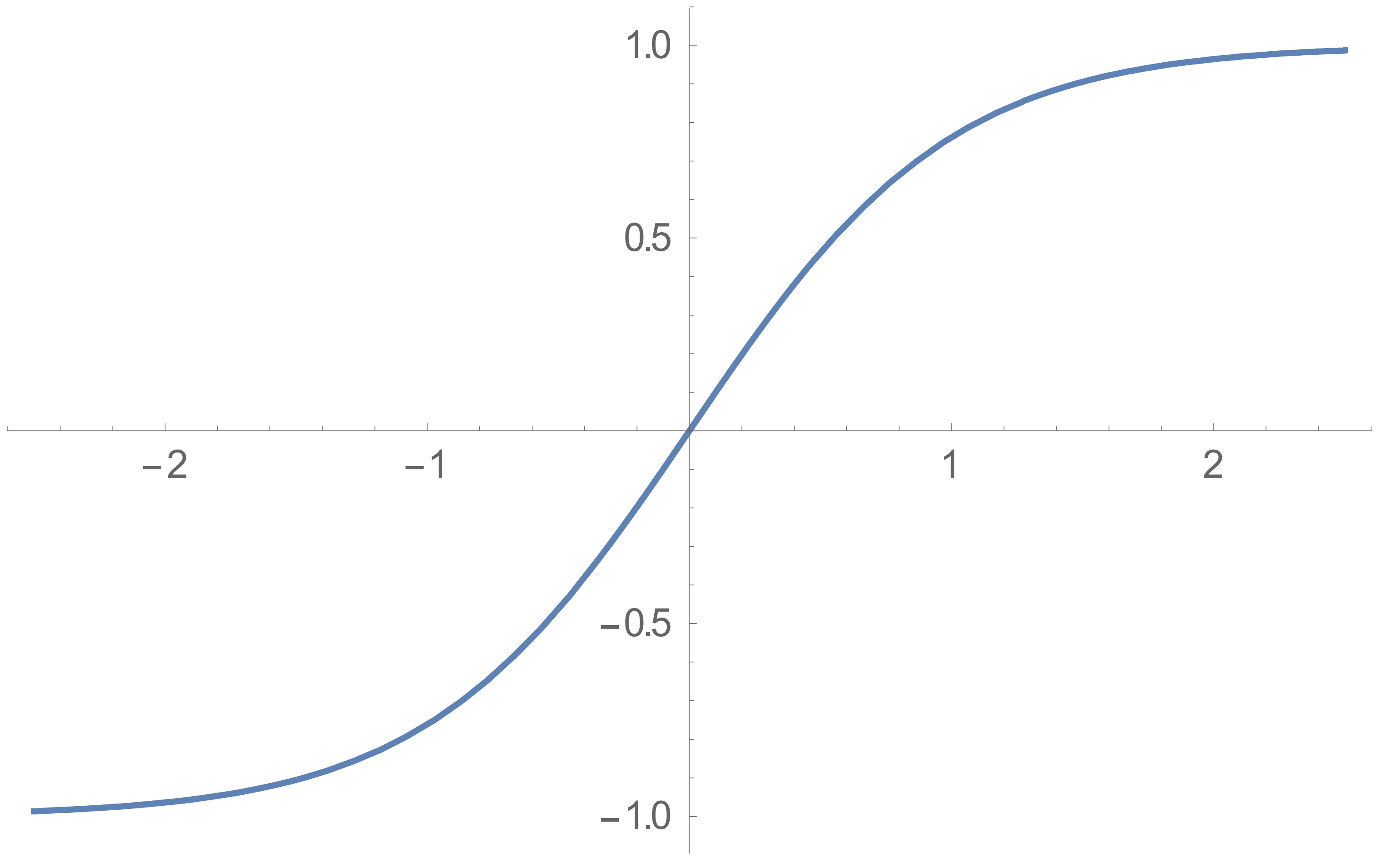
$$\chi = \frac{g^2 \mu_B^2}{k_B T} \left\langle (S_{tot}^z)^2 \right\rangle = \frac{g^2 \mu_B^2}{k_B T} \left(\sum_i \left\langle (S_i^z)^2 \right\rangle + \sum_{i \neq j} \left\langle S_i^z S_j^z \right\rangle \right)$$

Mean field theory

$$H = \sum_{n,m} J_{nm} \vec{S}_n \cdot \vec{S}_m - g\mu_B \sum_n B S_n^z$$

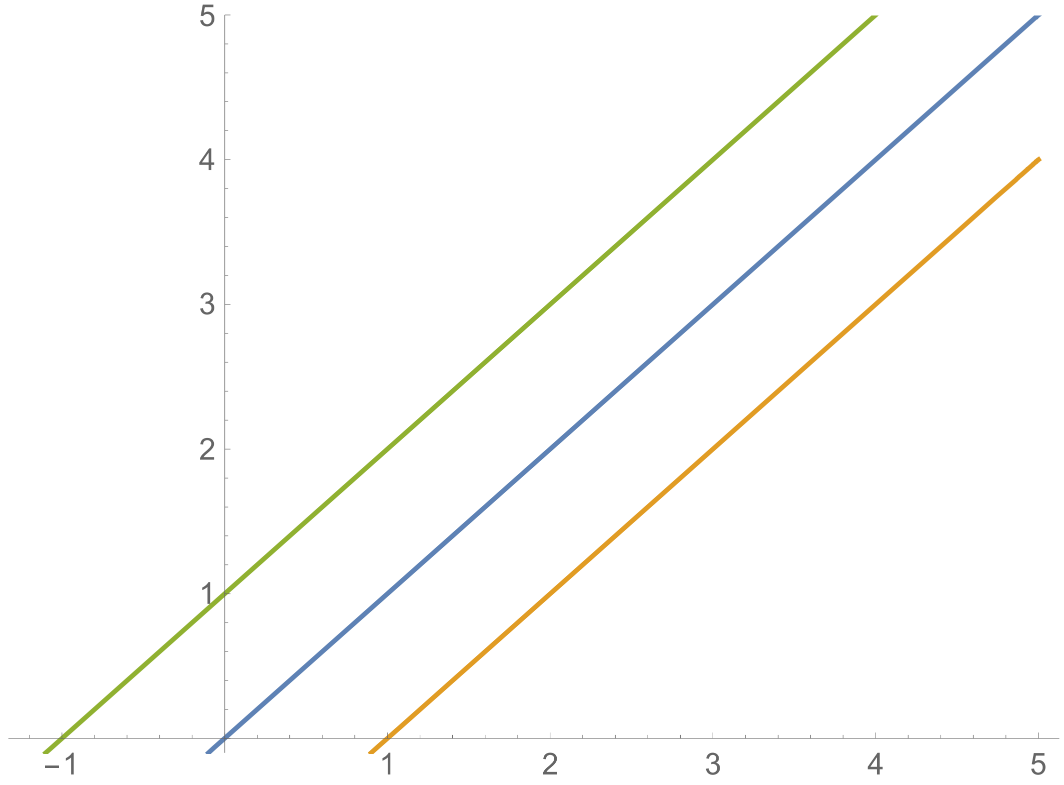
Self-consistency equation

$$\left\langle S_n^z \right\rangle = sB_s(s\beta g\mu_B B_{eff})$$



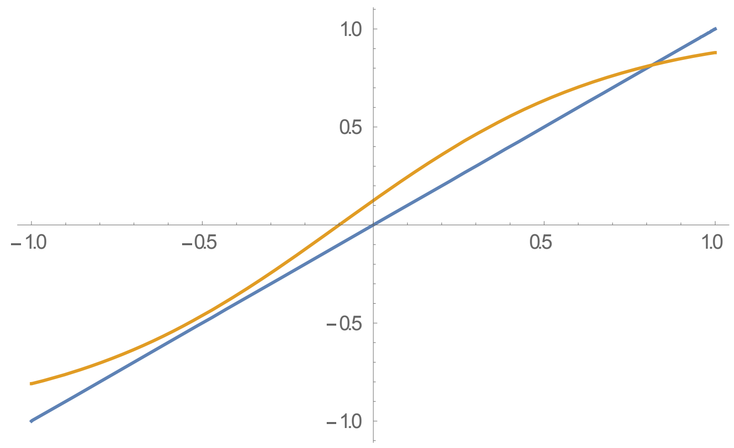
Mean field Susceptibility

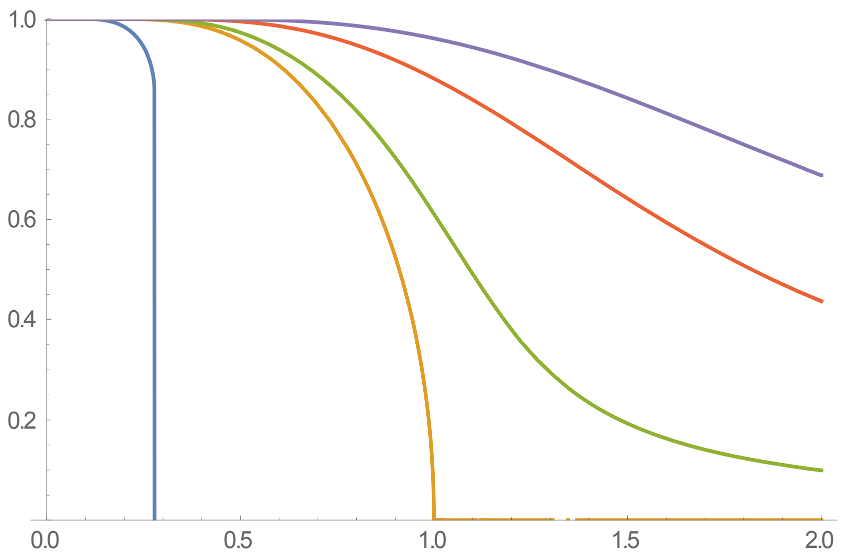
$$\chi = g\mu_B \frac{\partial \langle S_n^z \rangle}{\partial B}$$

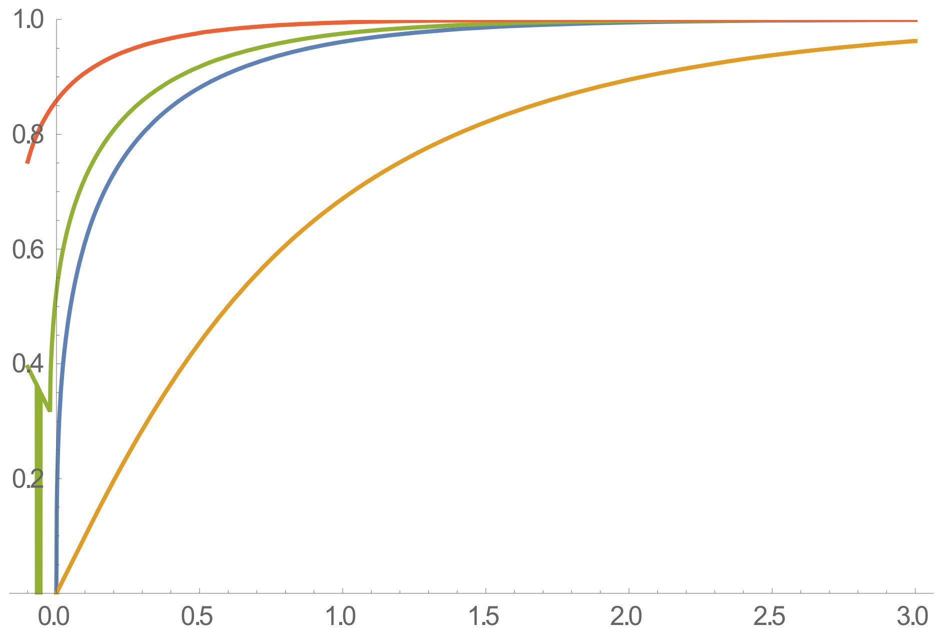


Discussion of solution for finite fields

$$\left\langle S_n^z \right\rangle = sB_s(s\beta g\mu_B B_{eff})$$







Cluster mean field theory: Bethe lattice

$$\chi = \frac{\chi_0}{1 + zJ\chi_0}$$