Magnetization and Susceptibility

Magnetization is a generalized force

$$M_z = -\left(\frac{\partial E}{\partial B_z}\right)_S = -\left(\frac{\partial F}{\partial B_z}\right)_T$$

$$Z = \sum_\lambda e^{-\beta \varepsilon_\lambda}$$

Susceptibility is a (linear) response

$$\chi_{\alpha\beta} = \frac{\partial M_\alpha}{\partial B_\beta}$$

Magnetism of a single particle

$$H = \frac{1}{2m}\left(\vec{P} - \frac{e}{c}\vec{A}\right)^2 - g\mu_B \vec{S}\cdot\vec{B} + V(\vec{R})$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

Paramagnetism: Magnetic moments

From linear term:

$$M_z = -\left(\frac{\partial F}{\partial B_z}\right)_T$$

$$Z = \sum_\lambda e^{-\beta \varepsilon_\lambda}$$

$$M_z = \mu_B \left\langle g S_z + L_z \right\rangle$$

Diamagnetism: Orbital movement

From quadratic term:

$$M_z = -\left(\frac{\partial F}{\partial B_z}\right)_T = -\frac{e^2}{8mc^2}\left(\frac{\partial \left\langle (\vec{R}\times\vec{B})^2 \right\rangle}{\partial B_z}\right)_T$$

Energy scales for localized orbitals

Hund's Rules for partially filled shells

1.) Maximize total spin by filling orbitals with spin up electrons, then spin down

2.) Maximize total angular momentum by occupying large Lz orbitals first

3.) Assume anti-ferromagnetic spin-orbital coupling for less than half filling, ferromagnetic otherwise

Wigner Eckart Theorem

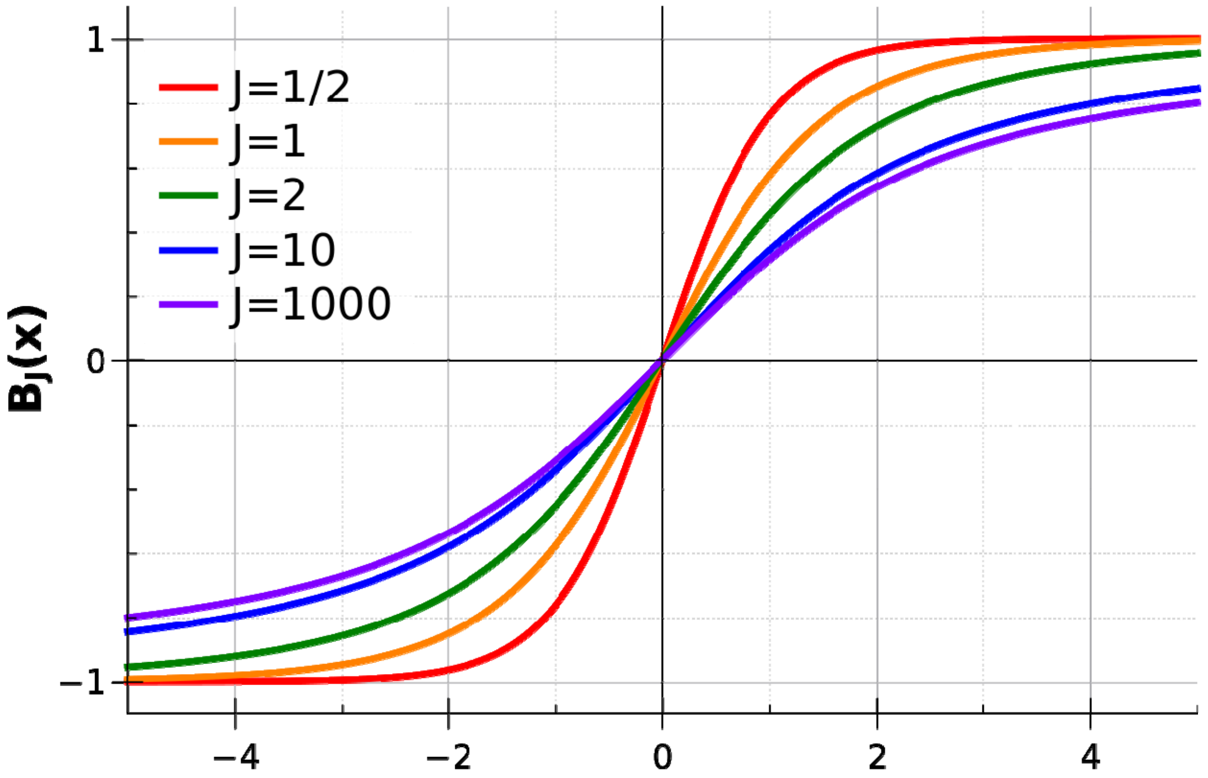
The matrix elements of a sum or angular momentum operators $-\mu_B(g\vec{S}+\vec{L})$

can be described by the total angular momentum operator $\vec{J}=\vec{S}+\vec{L}$

Magnetization of a single "spin"

$$H = -g\mu_B \vec{J} \cdot \vec{B}$$

$$Z = \sum_{\lambda} e^{-\beta\varepsilon_\lambda}$$

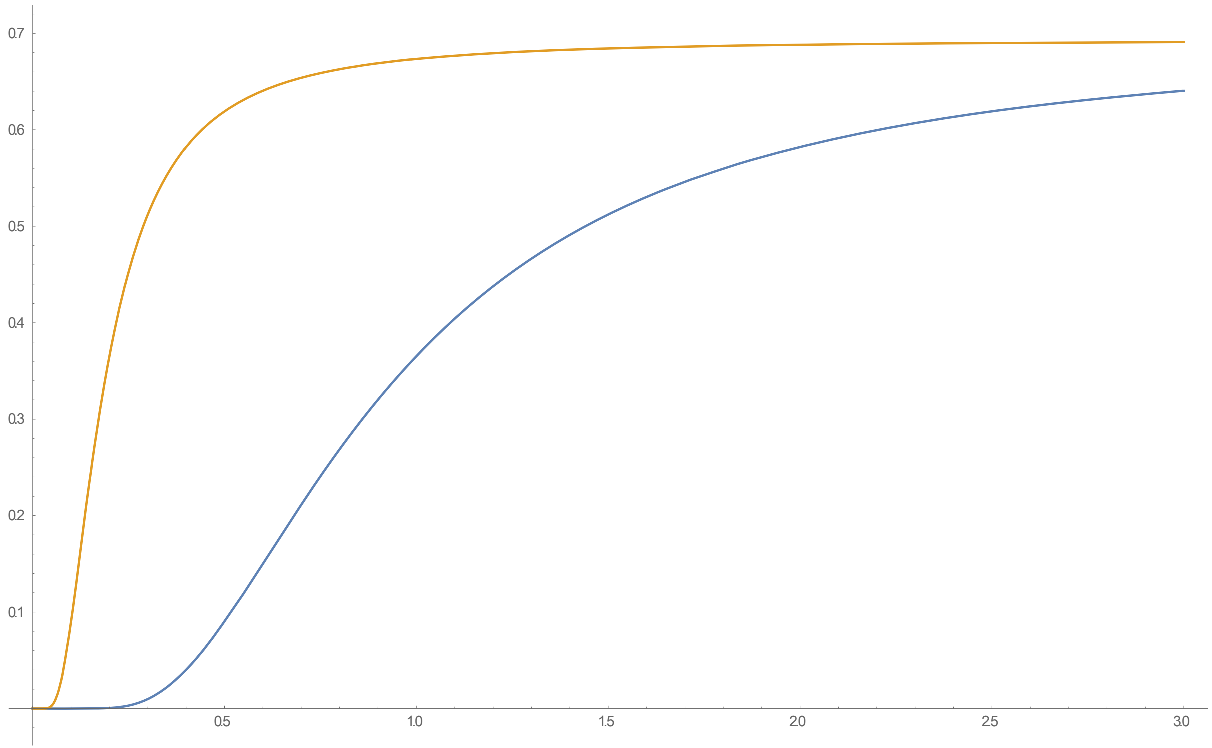


Curie Susceptibility

$$\chi = -\frac{\partial M}{\partial B}$$

Adiabatic demagnetization: cooling with magnetic fields

$$S = -\left(\frac{\partial F}{\partial T}\right)$$



Magnetization of coupled spins: Fluctuations

$$M_z = -\left(\frac{\partial F}{\partial B_z}\right)_T$$

Susceptibility of coupled spins: Correlations

$$\chi = \frac{g^2 \mu_B^2}{k_B T} \left\langle (S_{tot}^z)^2 \right\rangle$$

Coupled Spins

Hund's rules

Exchange and Superexchange

Dipolar Couplings

Heisenberg Model

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

XY-Model

$$H = \sum_{\langle i,j \rangle} J_{ij} (S_i^x \cdot S_j^x + S_i^y \cdot S_j^y)$$

Ising-Model

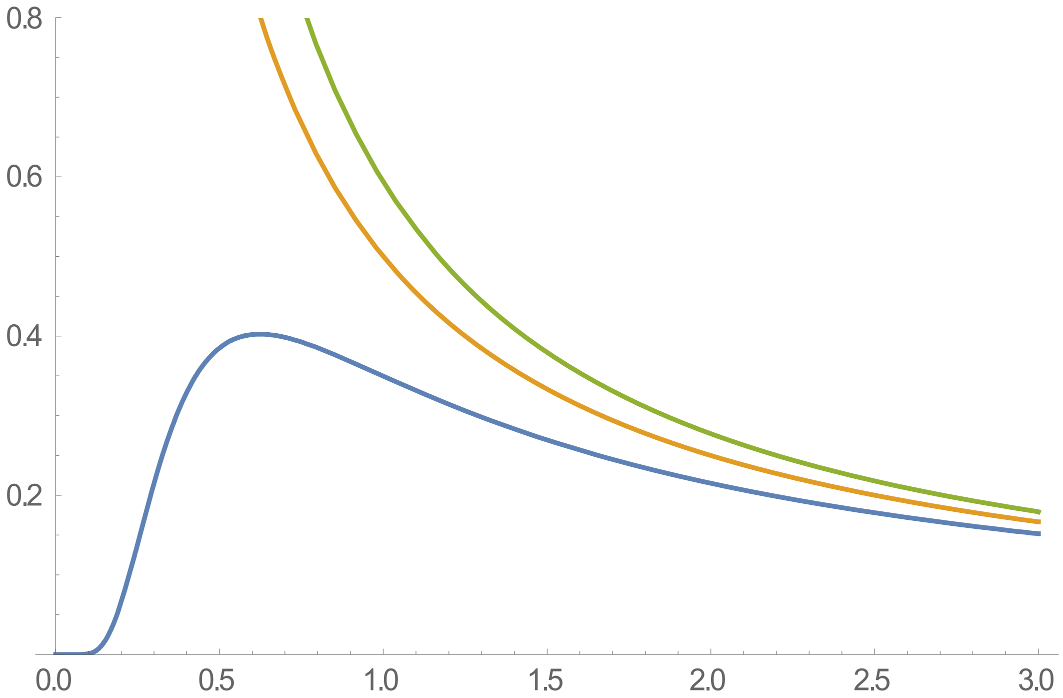
$$H = \sum_{\langle i,j \rangle} J_{ij} S_i^z \cdot S_j^z$$

Example: Two coupled Spins in a Field

$$H = J\vec{S}_1 \cdot \vec{S}_2 - B(S_1^z + S_2^z)$$

Dimer Susceptibility

$$\chi = \frac{g^2 \mu_B^2}{k_B T} \left\langle (S_{tot}^z)^2 \right\rangle$$



Ferromagnetic, Paramagnetic and Antiferromagnetic Susceptibilities