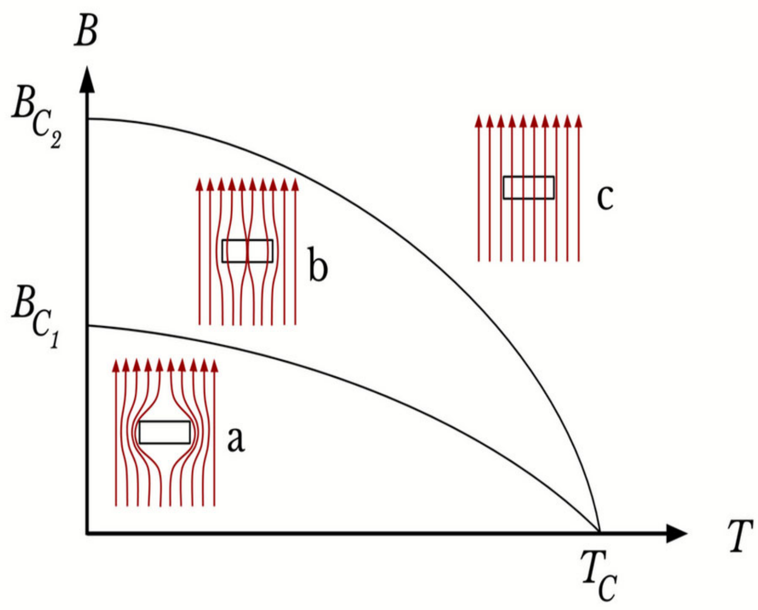
Summary of penetration depth and coherence length

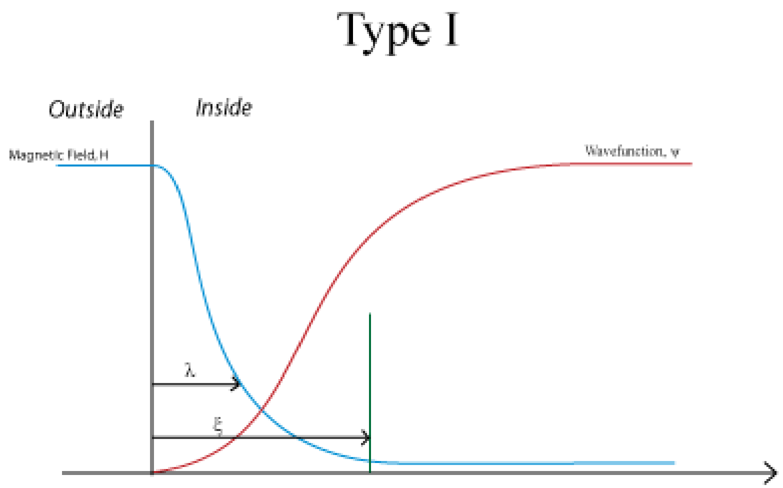
$$\lambda_L^2 = \frac{mc^2}{4\pi n_s e^2}$$

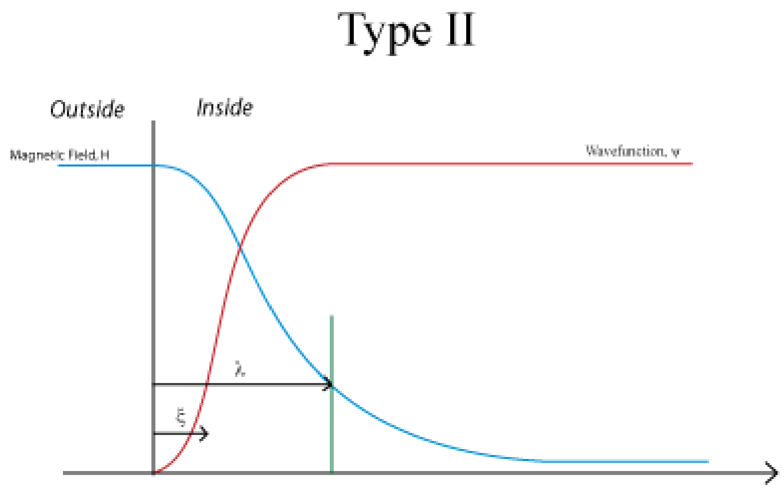
$$\xi^2 = \frac{2\pi^2\hbar^2 n_s}{mB_c^2}$$

Material	Coherence length ξ_0(nm)	London penetration depth λ_L(nm)	Ratio λ_L/ξ_0
Sn	230	34	0.16
Al	1600	16	0.010
Pb	83	37	0.45
Cd	760	110	0.14
Nb	38	39	1.02

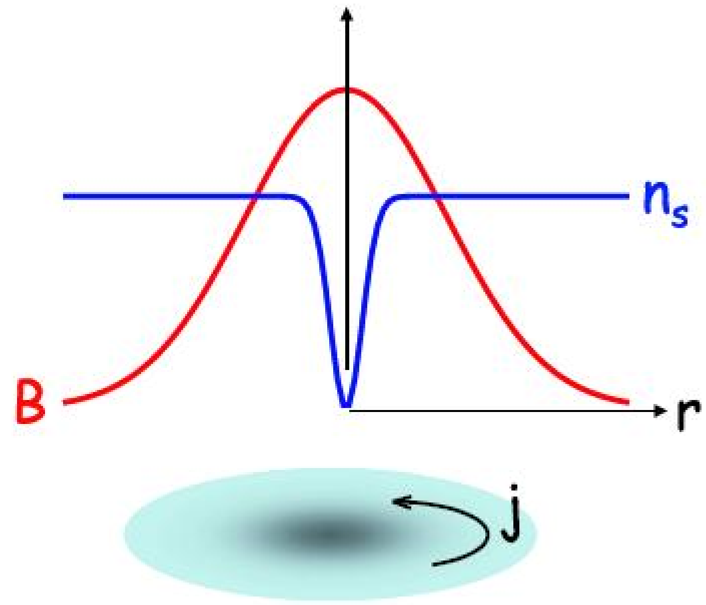
Type-I and Type II Superconductors







Abrikosov Vortices



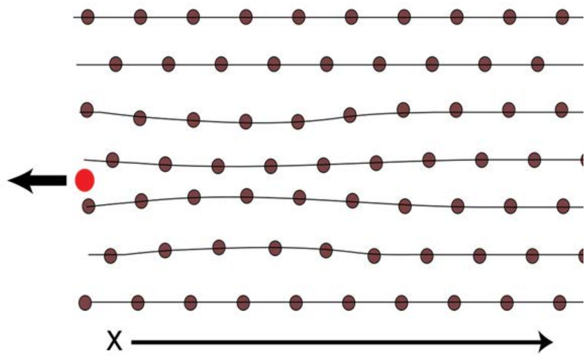
Electron-Phonon Interactions

$$\hat{H}_{el} = \sum_{\alpha,\vec{k}} \varepsilon_\alpha(\vec{k}) \hat{\Psi}^\dagger_{\vec{k},\alpha} \hat{\Psi}_{\vec{k},\alpha}$$

$$\hat{H}_{ph} = \sum_{\gamma,\vec{k}} \hbar\omega^\gamma_{\vec{k}} \left(\hat{a}^{\gamma\,\dagger}_{\vec{k}} \hat{a}^\gamma_{\vec{k}} + \tfrac{1}{2} \right)$$

Electron density couples to displacement field

$$H_{\text{el-ph}} = T \sum_j n_{el}(\vec{R}_j) D(\vec{R}_j)$$



Fourier expansion of the electron density

$$n_{el}(\vec{R}) = \sum_{\alpha} \widetilde{\Psi}^{\dagger}_{\vec{R},\alpha} \widetilde{\Psi}_{\vec{R},\alpha}$$

$$\widetilde{\Psi}_{\vec{R},\alpha}(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{k} \in 1BZ} e^{i\vec{k}\cdot\vec{R}} \Psi_{\vec{k},\alpha}$$

Fourier expansion of the displacement field

$$D(\vec{R}_j) = \vec{\nabla}_{\vec{R}_j} \cdot \vec{X}_j$$

$$X_j^{\alpha} = \frac{1}{\sqrt{N}} \sum_{\vec{k} \in 1BZ} e^{-i\vec{k}\cdot\vec{R}_j} X_{\vec{k}}^{\alpha} = \frac{1}{\sqrt{N}} \sum_{\vec{k} \in 1BZ} e^{-i\vec{k}\cdot\vec{R}_j} \sqrt{\frac{\hbar}{2m\omega_{\vec{k}}^{\alpha}}} \left(\hat{a}_{-\vec{k}}^{\alpha\,\dagger} + \hat{a}_{\vec{k}}^{\alpha} \right)$$

Electron-Phonon coupling for one band and one mode

$$H_{\text{el-ph}} = \widetilde{T} \sum_{j} n_{el}(\vec{R}_j) D(\vec{R}_j)$$

$$H_{\text{el-ph}} = \sum_{\mathbf{k},\mathbf{q}} T(\mathbf{q}) \psi^{\dagger}_{\mathbf{k}} \psi_{\mathbf{k}+\mathbf{q}} (a^{\dagger}_{\mathbf{q}} + a_{-\mathbf{q}})$$

Perturbation Theory: Derivation via Schrieffer-Wolff Transformation

General: $\hat{H} = \hat{H}^0 + \lambda \hat{H}^1$

Assume we have a few “intereresting” states $\hat{H}^0 \left| \alpha_0 \right\rangle = \varepsilon_\alpha^0 \left| \alpha_0 \right\rangle$ and the “rest” $\hat{H}^0 \left| \beta_0 \right\rangle = \varepsilon_\beta^0 \left| \beta_0 \right\rangle$

Goal: effective decoupling

Find a transformation with an operator $\hat{S}$ (or order λ) so that the two sectors decouple

$$\widetilde{H} = e^{-\hat{S}} \hat{H} e^{\hat{S}}$$

$$\widetilde{H} = (1 - \hat{S} + \frac{\hat{S}^2}{2} + \ldots)\hat{H}(1 + \hat{S} + \frac{\hat{S}^2}{2} + \ldots)$$

Decoupling condition:

$$0 = \widetilde{H}_{\alpha\beta} \approx \lambda \hat{H}^{1}_{\alpha\beta} + [\hat{H}_0, \hat{S}]_{\alpha\beta}$$

Effective Hamiltonian in decoupled sector:

$$\widetilde{H}_{\alpha\alpha} \approx$$

Phonon decoupling

$$\lambda H^1 = \sum_{\mathbf{k},\mathbf{q}} T(\mathbf{q}) \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}+\mathbf{q}} (a^\dagger_{\mathbf{q}} + a_{-\mathbf{q}}) = -[\hat{H}_0, \hat{S}]$$

$$\hat{H}^0 = \sum_{\vec{k}} \varepsilon(\vec{k}) \hat{\Psi}^\dagger_{\vec{k}} \hat{\Psi}_{\vec{k}} + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \left(\hat{a}^\dagger_{\vec{k}} \hat{a}_{\vec{k}} + \tfrac{1}{2} \right)$$

Ansatz:

$$S = \sum_{\mathbf{k}_1,\mathbf{q}_1} T(\mathbf{q}_1) \psi^\dagger_{\mathbf{k}_1} \psi_{\mathbf{k}_1+\mathbf{q}_1} (x\, a^\dagger_{\mathbf{q}_1} - y\, a_{-\mathbf{q}_1})$$

Determine S

$$\lambda H^1 = H_{\text{el-ph}} = \sum_{\mathbf{k},\mathbf{q}} T(\mathbf{q})\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}+\mathbf{q}}(a^\dagger_{\mathbf{q}} + a_{-\mathbf{q}}) = -[\hat{H}_0, \hat{S}]$$

Effective electron-electron interaction

$$H_{\text{eff}} = H^0 + \lambda H^1 + \frac{\lambda}{2}[H^1, S]$$