define a new field

$$(420) \qquad \boxed{\hat{\psi}(z,t) = e^{i\alpha\hat{\Theta}(z,t)}\,\hat{\Phi}(z,t)}$$

anyon field

where $\hat{\Theta}(z)$ is again the angle operator

$$(421) \qquad \hat{\Theta}(z) = \int d^2z'\, \varphi(z-z')\, \underbrace{\hat{\Phi}^{+}(z')\hat{\Phi}(z')}_{=\hat{n}(z')}$$

then following the same arguments as in Sec. 9.4A

$\Rightarrow$

$$(422) \qquad \boxed{\begin{aligned} \hat{\psi}(y)\,\hat{\psi}(x) &= e^{i\alpha\pi}\,\hat{\psi}(x)\,\hat{\psi}(y) \\ \hat{\psi}^{+}(y)\,\hat{\psi}(x) &= e^{i\alpha\pi}\,\hat{\psi}(x)\,\hat{\psi}^{+}(y) + \delta^{(2)}(x-y) \end{aligned}}$$

Note that different from Sec. 9.4A α is not necessarily an integer!

9.6* Effective Hamiltonian of quasi-holes: Anyons and Chern-Simon field

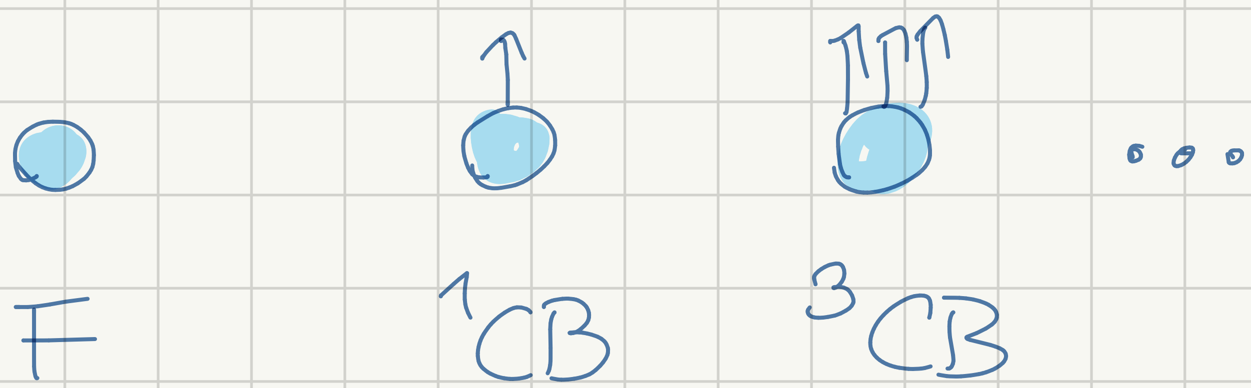
Now we want to sketch the derivation of an effective anyon Hamiltonian of quasiholes in the FQH system.

(A) FQHE as condensate of composite bosons

An alternative to the explanation of the FQHE as integer QHE of composite fermions in a reduced effective field is the interpretation in terms of composite bosons in a vanishing effective field.

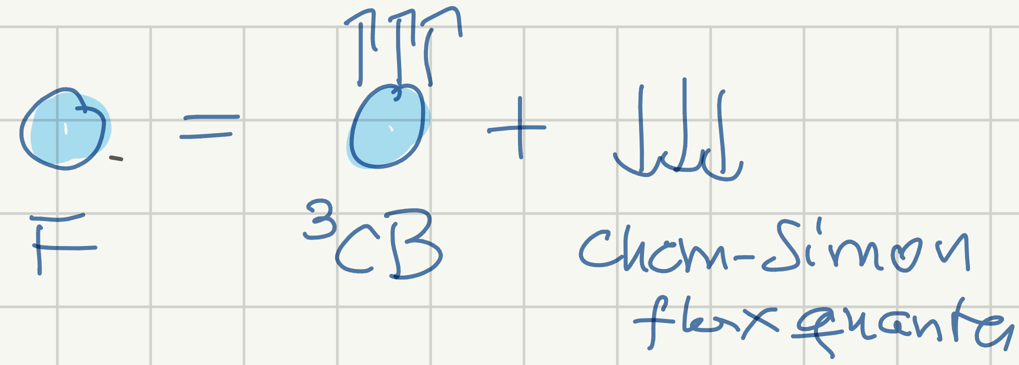
composite bosons

${}^{2p+1}CB$	boundstate of fermion and odd number $(2p+1)$ of flux quanta attached to them



effective magnetic field can be made to vanish

$$B^* = B - (2p+1)\varrho\,\Phi_0 = 0 \;\hat{=}\; \nu = \frac{1}{2p+1} \tag{423}$$



To construct the theory of quasi-holes it is more convenient to start from the composite-boson picture: $m = 2p+1$

(424)
$$H = \frac{1}{2M}\int d^2z\, \phi_m^+(z)\left(\tilde{P}_x + i\tilde{P}_y\right)\left(\tilde{P}_x - i\tilde{P}_y\right)\phi_m(z) + \frac{1}{2}N\hbar\omega_c$$

where

(425)
$$\left[\phi_m(z), \phi_m^+(z')\right] = \delta^{(2)}(z-z')$$

describes the composite-boson fields

$$\tilde{P}_\mu = -i\hbar\frac{\partial}{\partial x_\mu} + eA_\mu^{eff}$$

$$A_\mu^{eff} = A_\mu + C_\mu \qquad C_\mu \text{ Chern Simon field of } {}^{2p+1}CB$$

For $\nu = \frac{1}{2p+1}$

(426)
$$A_\mu^{eff} = 0$$

This is a convenient starting point to discuss quasiholes if $\nu = \frac{1}{2p+1} - \varepsilon$

(B) Quasiholes on top of a composite-boson fluid

To create N_{qh} quasiholes we start from a CB liquid with $A^{eff}=0$ and perform a singular gauge trafo

(427) $$\hat{\phi}_m \to e^{if}\hat{\phi}_m \qquad (m=2p+1)$$

(428) $$f=\sum_{r=1}^{N_{qh}} \hat{\Theta}(z-z_r)$$

where z_r are the positions of the quasi-holes. (427) will change the Chern-Simon field

(429) $$C_\mu \to C'_\mu = C_\mu - \frac{\hbar}{e}\partial_\mu f$$

Note that now $\nu = \frac{1}{2p+1} - \varepsilon$

(430) $$A_\mu^{eff} = \underbrace{A_\mu + C_\mu}_{\neq 0 \text{ since } \nu \neq \frac{1}{2p+1}} - \frac{\hbar}{e}\partial_\mu f \overset{!}{=} 0 \text{ on average}$$

The number N_{qh} of quasiholes created is such that the effective magnetic field including the flux quanta associated to the quasiholes vanishes after averaging.

The new Chern Simon magnetic field is now

(431) $$\boxed{B_{CS} \to B_{CS} = m\Phi_0 n(z) + \Phi_0 n_{qh}(z)}$$

instead of (399), where $(m = 2p+1)$

(432) $$n_{qh}(z) = \sum_{r=1}^{N_{qh}} \delta^{(2)}(z - z_r(t))$$

is the density of quasi-holes which have positions $z_r(t)$.

Allowing for a kinetic energy of quasiholes with some (yet unknown but at the same time irrelevant) mass M_{qh} one can derive an effective Hamiltonian of quasiholes with

effective charge $\quad q^* = -\frac{1}{m} e$

exchange phase $\quad \frac{\pi}{m}$

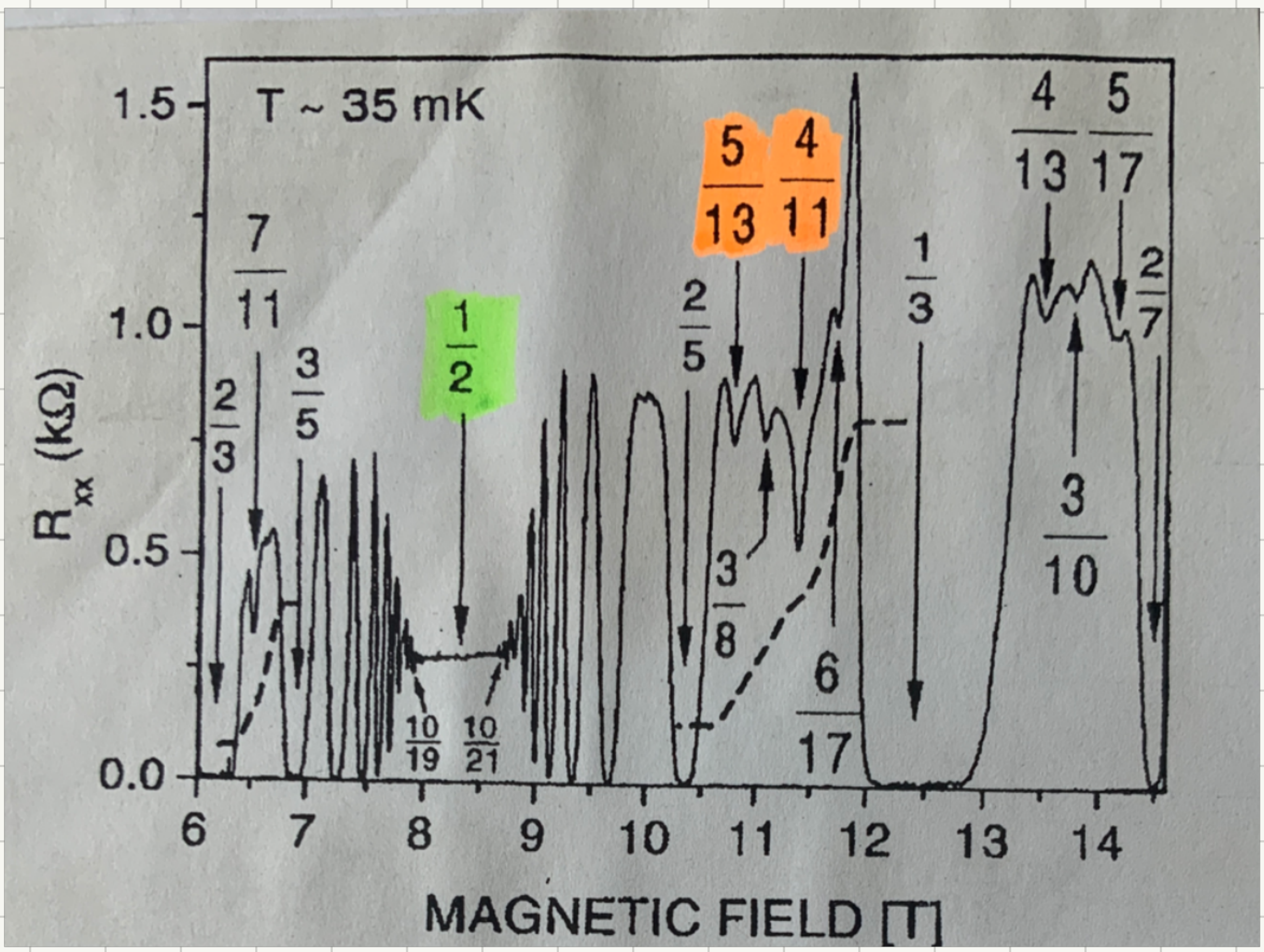
...

9.7 Interacting composite fermions

We have seen that the CF theory explains the existence of incompressible states at fillings

$$\nu = \frac{n}{2pn \pm 1} \tag{433}$$

with Hall plateaus. But there are also other fillings where Hall plateaus can be observed



e.g. at fillings $\nu = \frac{5}{13}$ or $\nu = \frac{4}{11}$

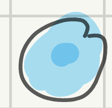
$\Rightarrow$ interactions between CF!

(A) higher FQH series

Pseudopotentials are not completely screened by the formation of composite fermions. So one can discuss the fractionalization of CF rather than of bare fermions

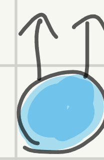
fermions at filling

$$\frac{1}{3} < \nu = \frac{\nu^*}{2\nu^*+1} < \frac{2}{5}$$



can be mapped to ^{2}CF with effective filling

$$1 < \nu^* = 1 + \underbrace{\frac{f}{2f\pm1}}_{} < 2$$



$\uparrow$ full lowest Λ level

$\Rightarrow$ FQHE of ^{2}CF with fillings

$f=1 \qquad \nu^* = 1 + \frac{1}{3} \quad \Rightarrow \quad \nu = \frac{4}{11}$

$f=2 \qquad \nu^* = 1 + \frac{2}{3} \quad \Rightarrow \quad \nu = \frac{5}{13}$

(B) Absence of FQH state at $\nu = \frac{1}{2}$

At $\nu = \frac{1}{2}$ there is no dip in $R_{\parallel}$ but the state is still interesting

$$(434) \qquad \nu = \frac{1}{2} = \lim_{n \to \infty} \frac{n}{2n \pm 1} \qquad (p=1)$$

i.e. ^2CF with filling

$$(435) \qquad n = \frac{g}{B_{eff}\Phi_0} = \infty \qquad \boxed{B_{eff} = 0}$$

These are free ^2CF without magnetic field which form a Fermi sea

(C) Exotic states at $\nu = 5/2$: CF-pairing

Willett et al. (PRL $\underline{59}$, 1776 (1987)) have seen a FQH plateau at $\nu = 5/2$. Here the Zeeman splitting is small and one has to consider both spin components

$$\nu = \frac{5}{2} = \underset{\substack{\uparrow \\ \text{lowest LL} \\ \text{for 2 spins}}}{2} + \underset{\substack{\nwarrow \\ \text{2nd LL} \\ \text{1 spin}}}{\frac{1}{2}}$$

$\Rightarrow$ Fermi sea of ^{2}CF in 2nd LL ?

no!

Moore & Read (Nucl. Phys. B360, 362 (1991)) have shown that a good candidate for the $\nu = \frac{5}{2}$ FQH state is a p-wave paired state of spin-polarized electrons in the 2nd LL. "Pfaffian" state

remark: • pairing of (composite) fermions requires attractive interaction

in 2nd LL ^{2}CF do indeed attract each other

• nature of $\nu = \frac{5}{2}$ state is still not fully understood.