i.e.

(563) $$|\psi\rangle \longrightarrow |\psi'\rangle = R(t)|\psi\rangle$$

and

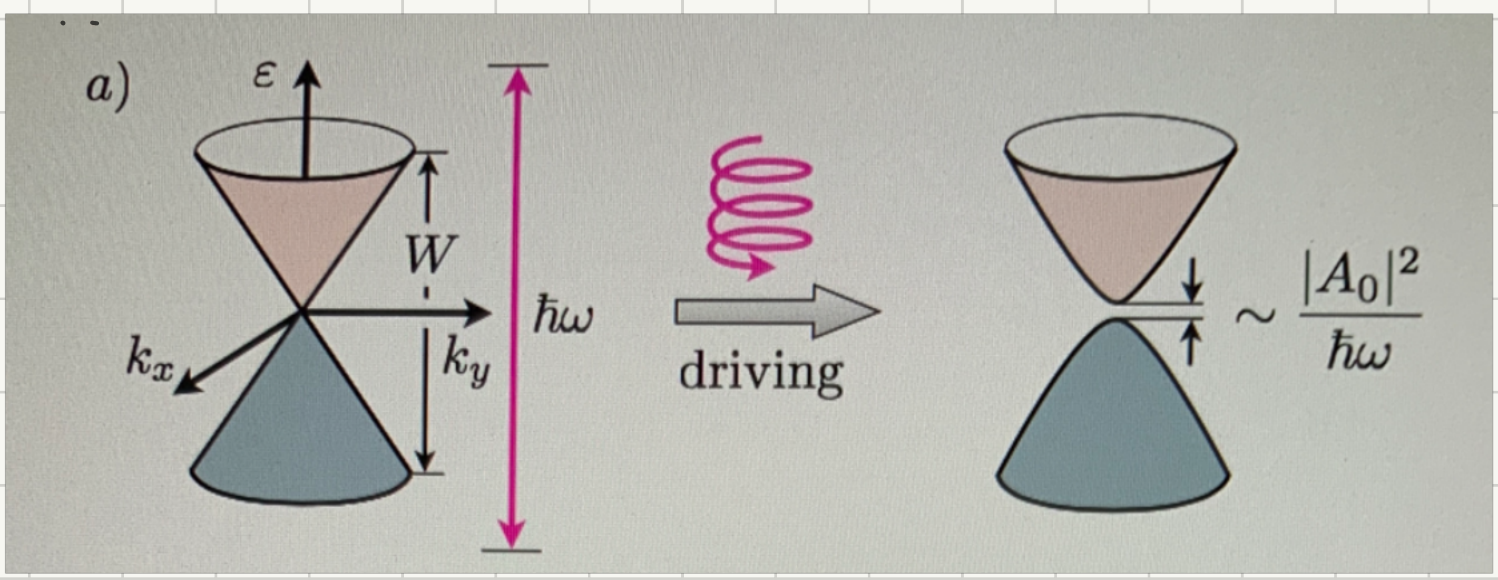
(564) $$\boxed{H(t) \rightarrow \tilde{H}(t) = RHR^{\dagger} - iR\frac{d}{dt}R^{\dagger}}$$

In $\tilde{H}$ we now got rid of terms $\sim\omega$ so we can apply the technique from Sec. (B).

12.2 Topology of Floquet bands

(A) Creating topological phases by high-frequency modulation

- Graphene irradiated by circular polarized light



low-energy Hamiltonian of Graphene

(565) $$\underline{h_0}(\vec{k},t) = v_F(\sigma_x \tau_x q_x + \sigma_y q_y)$$

where $\vec{q} = \vec{k} - \xi \vec{K}$ $\quad \xi = \pm 1$

σ - sublattice $\quad \tau$ - valley $K \leftrightarrow K'$

(566) with elm. driving

$$\underline{\underline{h}}(\vec{k},t) = v_F \left(\sigma_x \tau_x (q_x + eA_x(t)) + \sigma_y \quad (q_y + eA_y(t)) \right)$$

where

(567) $$\vec{A} = -A_0 \left(\sin\omega t \, \hat{e}_x + \sin(\omega t - \varphi) \hat{e}_y \right)$$

φ allows to tune polarization.

high-frequency limit $\quad \omega \gg eA_0 v_F$

(568) $$\underline{\underline{h}}_F(\vec{k}) \approx \underline{\underline{h}}_0 + \frac{1}{\omega} \left[\underline{\underline{h}}^{(-1)}, \underline{\underline{h}}^{(+1)} \right]$$

$$= \underline{\underline{h}}_0 \underbrace{- \frac{(eA_0 v_F)^2}{\omega} \sin\varphi \, \sigma_z \tau_z}_{\text{Haldane mass term}}$$

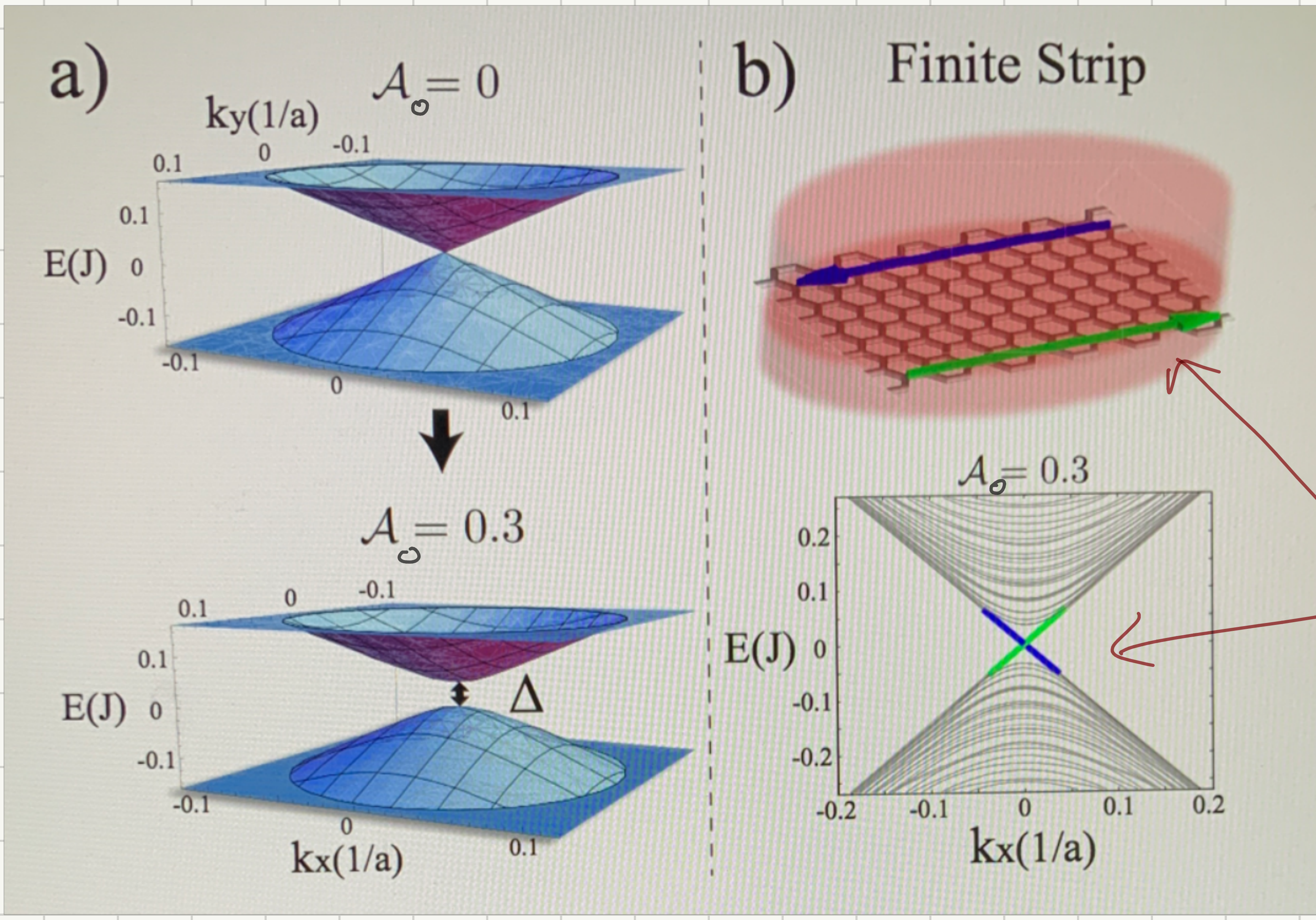
$\Rightarrow$ photoinduced gap $\quad (\hbar = 1)$

(569) $$\Delta = 2 \frac{(eA_0 v_F)^2}{\omega} \sin\varphi$$

vanishes for linear polarization $\varphi = 0, 2\pi$

chirality generated by circular polarized light!

$\Rightarrow$ Floquet topological insulator

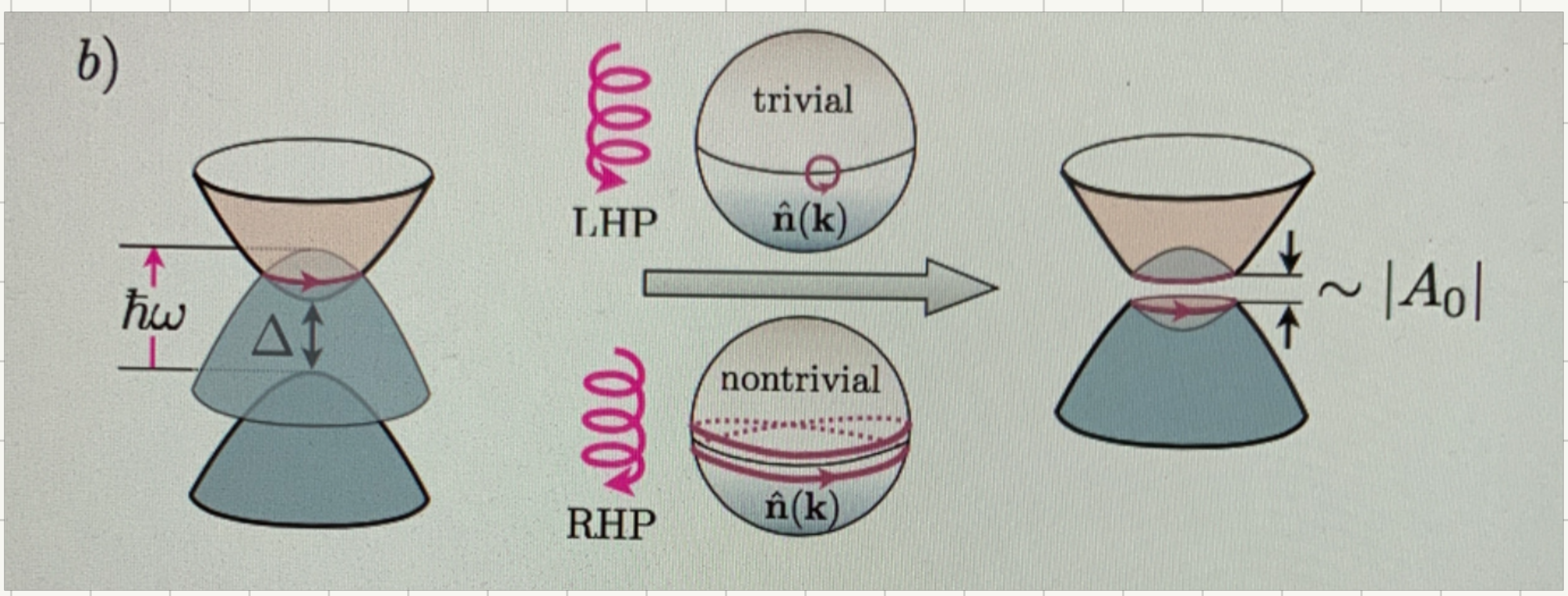


Kitagawa et al. PRB $\underline{84}$ 235108 (2011)

chiral edge modes

- HgTe/CdTe heterostructures with circular pol. light

A second example where fast periodic drive creates a topological insulator are HgTe/CdTe heterostructures



This system can show a QSHE (Bernevig, Hughes, Zhang Science $\underline{314}$, 1757 (2006)) and is TR symmetric

(570) $$\underline{\underline{h}}(\vec{k}) = \begin{pmatrix} \underline{\underline{\tilde{h}}}(\bar{k}) & 0 \\ 0 & \underline{\underline{\tilde{h}}}^*(-\bar{k}) \end{pmatrix}$$ 4×4

(571) $$\underline{\underline{\tilde{h}}}(\bar{k}) = \varepsilon(\vec{k})\mathbb{1} + \vec{d}(\bar{k})\cdot\underline{\underline{\vec{\sigma}}}$$ sub-block Hamiltonian

$\vec{d}(\bar{k})$ describes spin-orbit coupling. The spectrum has two doubly-degenerate subbands

(572) $$\varepsilon_\pm(\vec{k}) = \varepsilon(\vec{k}) \pm |\vec{d}(\bar{k})|$$ $\hat{d} = \frac{\vec{d}}{|\vec{d}|}$

Within each of the two sub-blocks of $\underline{\underline{h}}(\vec{k})$ one finds (opposite) Chern numbers

(573) $$C_\pm = \pm\frac{1}{4\pi}\int d^2\vec{k}\left(\frac{\partial\hat{d}}{\partial k_x}\times\frac{\partial\hat{d}}{\partial k_y}\right)\cdot\hat{d}$$

around the Γ-point

(574) $$\vec{d}(\vec{k}) \simeq (Ak_x, Ak_y, M - Bk^2)$$

where $A<0$, $B>0$ and M depend on the thickness of the quantum well. One can show that for the simplified band structure with (574)

(575) $$C_\pm = \pm\left[1 + \mathrm{sign}\left(\frac{M}{B}\right)\right]/2$$

So changing the sign of M modifies the sub-band Chern number by 1.

Note that $C_\pm + \bar{C}_\pm = 0$ by TR symmetry

- Start in topological trivial state

(576) $$M < 0 \qquad C_\pm = 0$$

Lindner, Refael, Galitski, Nature Phys **7**, 490 (2011):

$\Rightarrow$ topological phase transition from $M<0$ to $M>0$ by periodic drive:

(576) $$V = \vec{V} \cdot \underline{\underline{\vec{\sigma}}} \cos(\omega t)$$

To conserve TR symmetry: linear polarized light.

- Effective Floquet Hamiltonian for sub-band

$$\underline{\underline{\tilde{h}}} \rightarrow \underline{\underline{\tilde{h}}}^F(\vec{k}) = \varepsilon_F(\vec{k}) \mathbb{1}_{2\times 2} + \vec{n}(\vec{k}) \cdot \underline{\underline{\vec{\sigma}}} \qquad \hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

Floquet Chern number of sub-band

(577) $$C_\pm^F = \pm \frac{1}{4\pi} \int d^2k \left(\frac{\partial \hat{n}}{\partial k_x} \times \frac{\partial \hat{n}}{\partial k_y} \right) \cdot \hat{n}$$

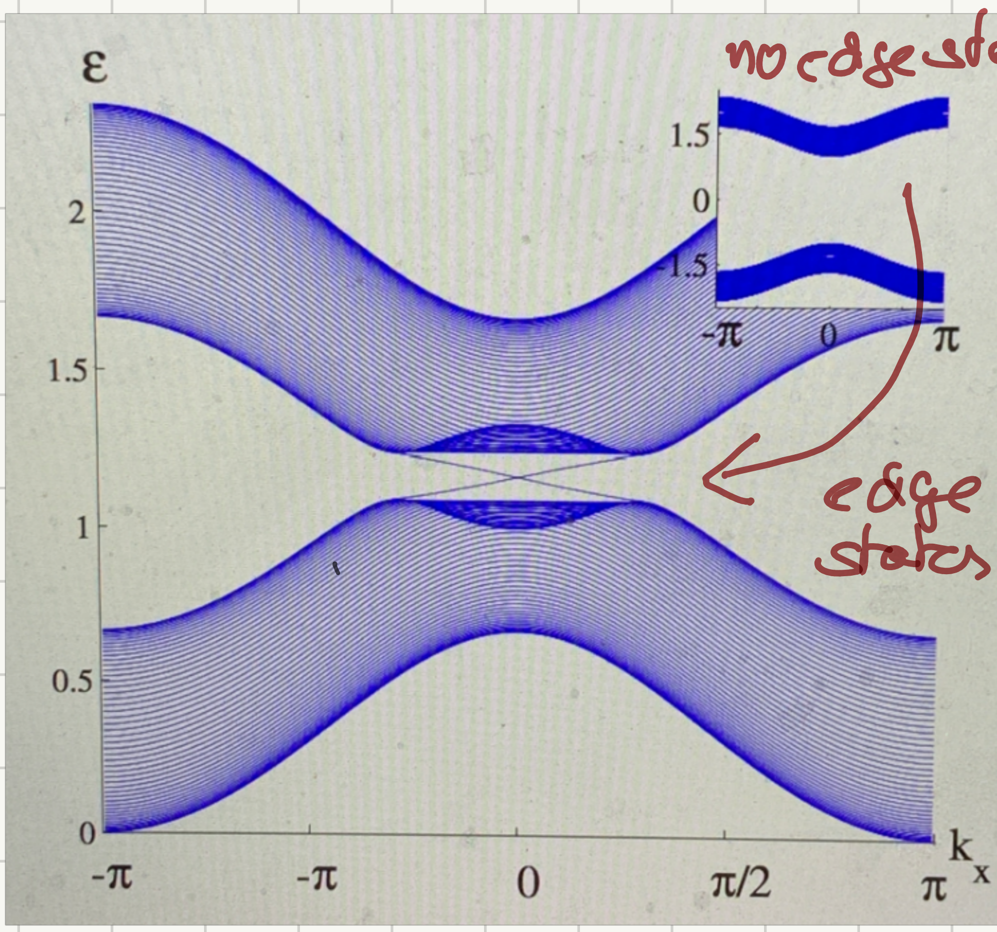
and

$\Rightarrow$

when starting from $M<0$

$$\boxed{C_\pm^F = C_\pm \pm 1}$$

(578)



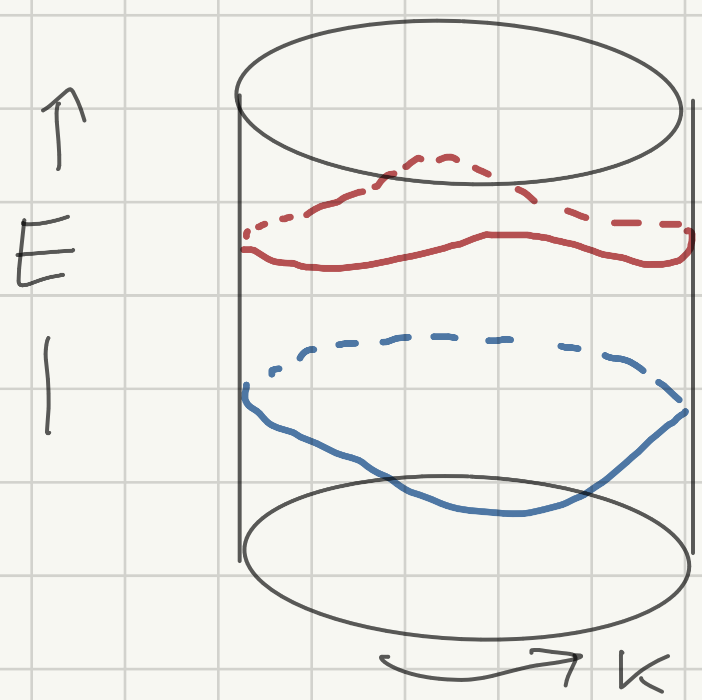
no edge states

edge states

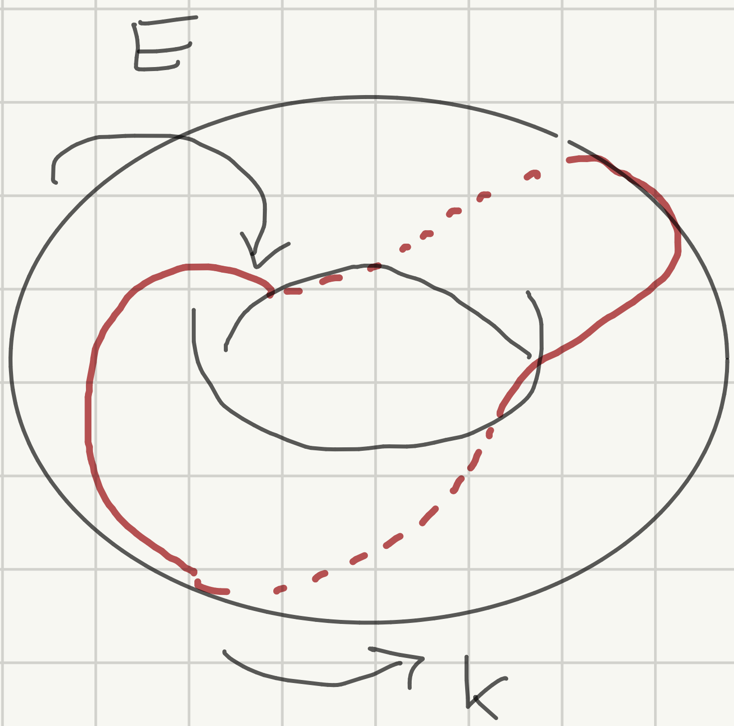
(B) Topology of Floquet bands beyond effective Hamiltonian

There are important generic cases where the Floquet Hamiltonian does not capture all topological properties. This is due to periodicity in quasi-energy

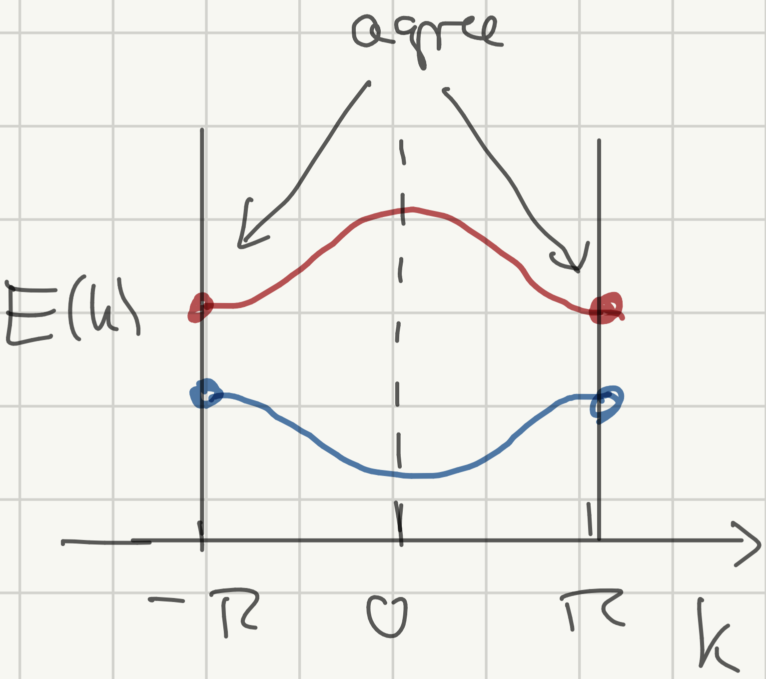
- there is no "top" or "bottom" in Floquet spectrum
- continuous band of quasienergies may wind around the Floquet Brillouin zone an integer number of times when the lattice momentum traverses the Brillouin zone.



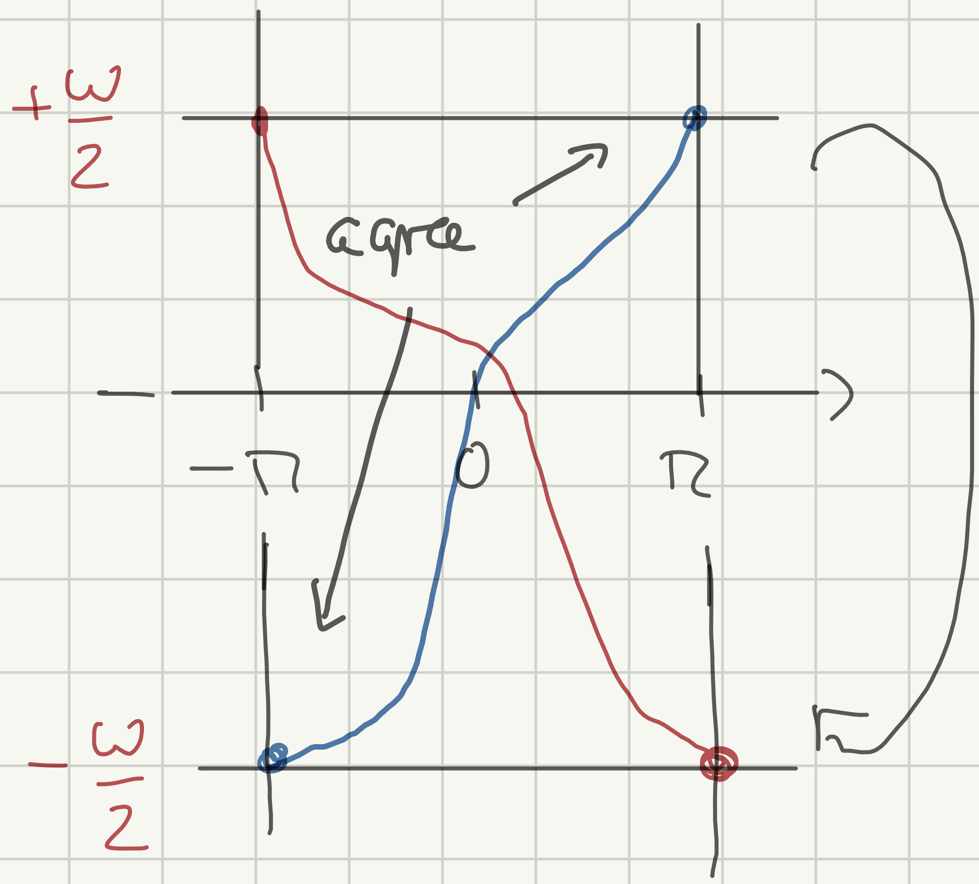
non-driven



Floquet



possible:



physical consequence of quasienergy winding

- quantized bulk transport in 1D Floquet system ($\hat{=}$ Thouless pump) in Floquet cycle

average group velocity of Floquet band

$$\bar{v}_g = \frac{a}{2\pi} \int_{BZ} dk \, \frac{\partial \varepsilon}{\partial k} = a \, \frac{W}{T} \tag{579}$$

a: lattice constant

$$\Rightarrow \quad \overline{\Delta x} = \bar{v}_g T = W a$$

In each cycle W units of charge are pumped

Floquet operator

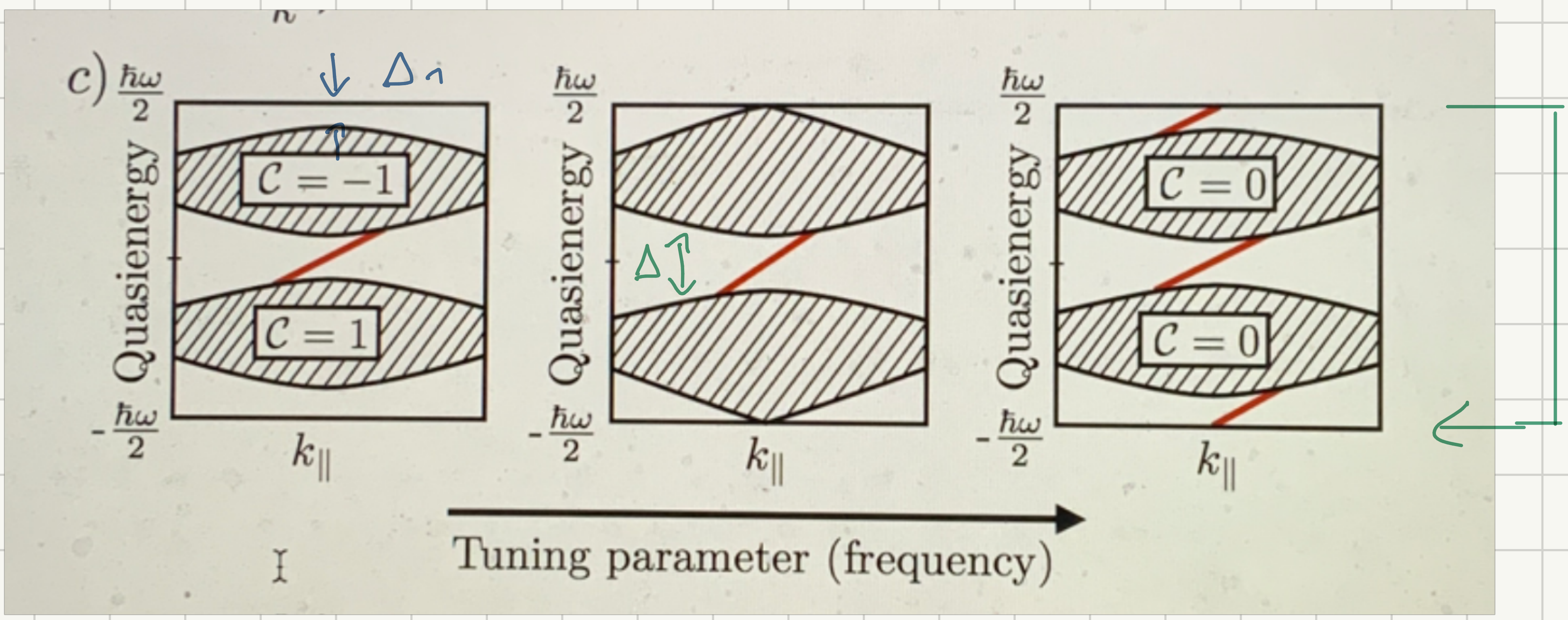
$$U_{1D}(k,T) = \mathcal{T} \exp\left(-i \int_0^T dt \, H_{1D}(k,t)\right)$$

(579) $$\boxed{\nu_1 = \frac{1}{2\pi}\int_{BZ} dk \; \mathrm{Tr}\left\{ U_{1D}^{\dagger}(k,T)\, i\frac{\partial}{\partial k} U_{1D}(k,T)\right\}}$$

GNVW index (Gross, Nesme, Vogts, Werner Comm. Math. physics 310, 419 (2012))

In general bands with opposite ν_1's which cross! Then transport vanishes. Hybridization gap $\sim 1/\omega$. I.e. for $\omega \to 0$ i.e. in the limit of slow i.e. quasi-adiabatic drive

- new topological transitions due to periodicity in quasi-energy



gap Δ never closes, so edge mode in the middle cannot disappear but quasienergy gap Δ_1 may close and

$$C = \pm 1 \longrightarrow C = 0$$

Spectral flow:

$$\boxed{\text{net chirality of edge modes above and below a band must be equal to Chern number of band}}$$

$\Rightarrow$ emergence of novel "edge" modes at top and bottom i.e. at $\varepsilon \approx \pm \omega/2$ in addition to that at $\varepsilon \approx 0$

anomalous edge states $\quad C_F = 0$

Potential problem:

Many body systems in Floquet systems can continuously absorb energy and heat up.
Combining periodic drive with reservoir couplings may overcome this problem