Please note: Exercises 18 and 19 are mandatory and have to be submitted to the postboxes in the 5th floor of building 46.

Exercise 18. Phonon gas
Consider again the phonon gas from exercise 17. Show that for small temperatures the equation

$$E = E_0(V) + C_1 T^4 \qquad (\textit{Debye law}), \tag{1}$$

i.e. $C_V \sim T^3$, and for large temperatures the equation

$$E = E_0(V) + 3Nk_{\mathrm{B}}T \qquad (\textit{Dulong–Petit law}), \tag{2}$$

i.e. $C_V \sim 3Nk_{\mathrm{B}}$, hold.
Hint: Make use of the formal analogy to the photon gas.

Exercise 19. Joule-Thomson effect
Consider a van der Waals gas with equation of state

$$\left(p + \frac{a}{v^2}\right)(v - b) = k_B T. \tag{3}$$

Calculate the Joule-Thomson coefficient $\alpha = \left(\dfrac{\partial T}{\partial p}\right)_{\mathrm{H}}$.
The inversion curve $p = p(v)$ is defined by

$$\alpha = \frac{1}{T}. \tag{4}$$

At which temperature T_{inv} is the Joule-Thomson coefficient always negative?