*Please note:* Exercises 16 and 17 are mandatory and have to be submitted to the postboxes in the 5th floor of building 46.

**Exercise** 16. Bose condensation in 1D and 2D
Calculate the particle density of excited states of a homogeneous, infinitely expanded ideal Bose gas at a given Temperature $T$ and chemical potential $\mu$ in one (1D) and two (2D) spatial dimensions. Is the density bounded from above? What are the consequences for Bose condensation in 1D and 2D?

**Exercise** 17. Phonon gas
Lattice vibrations in a solid can be described by a lattice of coupled harmonic oscillators. The classical Hamilton function of this system in one dimension is given by:

$$H = \sum_n \left( \frac{m}{2} \dot{y}_n^2 + \frac{K}{2} (y_n - y_{n-1})^2 \right), \tag{1}$$

where $y_n = x_n - x_n^0$ are the displacements from the resting positions $x_n^0$ and $a = x_{n+1}^0 - x_n^0$ is the lattice constant. Show that equation (1) can be reduced to a sum of harmonic oscillators that is equivalent to a quantum mechanical Hamiltonian:

$$\hat{H} = \sum_k \hbar\omega_k \left( \hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \right), \tag{2}$$

where:

$$\omega_k = 2\sqrt{\frac{K}{m}} \sin\left(\frac{ka}{2}\right), \quad -\frac{\pi}{2} \leq \frac{ka}{2} \leq \frac{\pi}{2}. \tag{3}$$

These excitations are called *acoustic phonons.*