Please note: Exercises 16 and 17 are mandatory and have to be submitted to the postboxes in the 5th floor of building 46.

Exercise 16. Bose condensation in 1D and 2D
Calculate the particle density of excited states of a homogeneous, infinitely expanded ideal Bose gas at a given temperature $T$ and chemical potential $\mu$ in one (1D) and two (2D) spatial dimensions. Is the density bounded from above? What are the consequences for Bose condensation in 1D and 2D?

Exercise 17. Phonon gas
Lattice vibrations in a solid can be described by a lattice of coupled harmonic oscillators. The classical Hamilton function of this system in one dimension is given by:

$$H = \sum_n \left( \frac{m}{2} \dot{y}_n^2 + \frac{K}{2} (y_n - y_{n-1})^2 \right),$$

where $y_n = x_n - x_0^n$ are the displacements from the resting positions $x_0^n$ and $a = x_0^{n+1} - x_0^n$ is the lattice constant. Show that equation (1) can be reduced to a sum of harmonic oscillators that is equivalent to a quantum mechanical Hamiltonian:

$$\hat{H} = \sum_k \hbar \omega_k \left( \hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \right),$$

where:

$$\omega_k = 2 \sqrt{\frac{K}{m}} \sin \left( \frac{ka}{2} \right), \quad -\frac{\pi}{2} \leq \frac{ka}{2} \leq \frac{\pi}{2}.$$ 

These excitations are called acoustic phonons.