Please note: Exercises 14 and 15 are mandatory and have to be submitted to the postboxes in the 5th floor of building 46.

Exercise 14.
Consider a one-dimensional gas of electrons \( S = 1/2 \) with \( N \) particles in \((0, L)\).

(a) What are the Fermi momentum \( p_F \) and Fermi energy \( \epsilon_F \)?

(b) Calculate \( \mu = \mu(T, N/L) \) analogously to the lecture.

Exercise 15.
Calculate the particle-number fluctuation \( \Delta n_p^2 \) of an ideal quantum gas (bosons and fermions) in the grand canonical ensemble at temperature \( T \) and in momentum state \( p \). Express them via the expectation value \( \langle n_p \rangle \). What is the qualitative difference between bosons and fermions?