Note: Please insert your assignment into the mailboxes 5th floor, building 46. The problems marked by a ☗ have to be submitted.

☗ **Problem 33.** *Radial component of the momentum*
Show that for the operators of distance to the origin $\hat{r}$ and the radial component of the momentum $\hat{p}_r$ hold:

$$[\hat{r}, \hat{p}_r] = i\hbar \tag{1}$$

$$[\hat{p}_r, \hat{\vec{L}}^2] = 0. \tag{2}$$

☗ **Problem 34.** *Quantum dot*
A spherical semiconductor quantum dot can be described as a free electron in a spherical symmetric potential

$$V(r) = \begin{cases} 0 & \text{for } r \leq R \\ \infty & \text{for } r > R \end{cases}. \tag{3}$$

Determine the stationary states of the single-particle Schrödinger equation in the potential $V(r)$ and the corresponding energies. What is the degree of degeneracy of the eigenstates?

Problem 35. *Spherical harmonic oscillator*
Consider the three-dimensional isotropic harmonic oscillator

$$V(r) = \frac{1}{2} m\omega^2 r^2. \tag{4}$$

Determine the eigenvalues and functions (without normalization) in spherical coordinates. Use the ansatz

$$\phi_E(\vec{r}) = \frac{u_l(r)}{r} Y_l^m(\vartheta, \varphi). \tag{5}$$

Introduce normalized coordinates $y = r/b$ and $E = \epsilon\hbar\omega$, where $b^2 = \hbar/m\omega$, and use

$$u_l = y^{l+1} v(\rho) \exp\left(-\frac{y^2}{2}\right), \qquad \rho = y^2, \tag{6}$$

which results from the asymptotic behaviour of stationary wave functions. For $v(\rho)$ you should obtain the associated Laguerre polynomials.

Problem 36. *Runge-Lenz vector*
Show that the Runge-Lenz vector

$$\hat{\vec{F}} = \frac{1}{2m}\left(\hat{\vec{p}} \times \hat{\vec{L}} - \hat{\vec{L}} \times \hat{\vec{p}}\right) - \frac{\alpha}{\hat{r}} \hat{\vec{r}} \tag{7}$$

commutes with the Hamilton operator $\hat{H} = \hat{\vec{p}}^2/2m - \alpha/r$ of the coulomb potential.