Note: Please insert your assignment into the mailboxes 5th floor, building 46. The problems marked by a ☗ have to be submitted.

☗ **Problem 17.** *Scattering from a box potential*
Consider a particle with mass m moving along the x-axis in a potential

$$V(x) = \begin{cases} 0 & \text{for } |x| > a \\ V_0 & \text{for } |x| \leq a. \end{cases}$$

A particle with energy $E > 0$ moves from $x = -\infty$ until it reaches the barrier. It has the wavefunction

$$\phi(x) = \begin{cases} e^{ikx} + re^{-ikx} & \text{for } x < -a \\ \beta_+ e^{Kx} + \beta_- e^{-Kx} & \text{for } -a \leq x \leq a \\ te^{ikx} & \text{for } x > a. \end{cases}$$

(a) Determine the probability current of the incident, the reflected and the transmitted part of the wavefunction.

(b) What is the reflection factor and transmission factor for $0 < E < V_0$ and for $E > V_0$

$$R := \left| \frac{j_{\text{refl}}}{j_0} \right|, \quad T := \left| \frac{j_{\text{trans}}}{j_0} \right| \quad ?$$

(c) Determine T for $E = V_0$?

☗ **Problem 18.** *Baker-Hausdorff-Theorem*

(a) Show that for arbitrary operators $\hat{A}$ and $\hat{B}$ and a complex number x hold:

$$e^{x\hat{A}} \hat{B} e^{-x\hat{A}} = \hat{B} + \left[\hat{A}, \hat{B}\right] x + \frac{1}{2!} \left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right] x^2 + \dots . \tag{1}$$

Hint: Consider the taylor series of the function $\hat{f}(x) = e^{x\hat{A}} \hat{B} e^{-x\hat{A}}$

(b) Use the equation (1) to proof the Baker-Hausdorff-Theorem

$$e^{\hat{A}} e^{\hat{B}} = e^{\hat{A} + \hat{B} + \frac{1}{2}\left[\hat{A}, \hat{B}\right]} = e^{\hat{A} + \hat{B}} e^{\frac{1}{2}\left[\hat{A}, \hat{B}\right]}, \tag{2}$$

if $\left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right] = \left[\hat{B}, \left[\hat{A}, \hat{B}\right]\right] = 0$.

Hint: Consider the function $\hat{f}(x) = e^{x\hat{A}} e^{x\hat{B}}$ and find a differential equation of first order by differentation of $\hat{f}$ and use of $e^{-x\hat{A}} e^{x\hat{A}} = \mathbb{1}$.

Problem 19. *Double-Delta-Potential*
Consider two identical attractive delta potentials at distance d:

$$V(x) = -F\left(\delta(x - \frac{d}{2}) + \delta(x + \frac{d}{2})\right),$$

where $F > 0$. Solve the stationary Schrödinger equation in the three intervals $-\infty < x \leq -d/2$, $-d/2 \leq x \leq d/2$, $d/2 \leq x < \infty$. Find all bound states and the corresponding energies.