Problem 17. Scattering from a box potential
Consider a particle with mass $m$ moving along the x-axis in a potential

$$V(x) = \begin{cases} 
0 & \text{for } |x| > a \\
V_0 & \text{for } |x| \leq a.
\end{cases}$$

A particle with energy $E > 0$ moves from $x = -\infty$ until it reaches the barrier. It has the wavefunction

$$\phi(x) = \begin{cases} 
e^{ikx} + re^{-ikx} & \text{for } x < -a \\
\beta^+e^{Kx} + \beta^-e^{-Kx} & \text{for } -a \leq x \leq a \\
te^{ikx} & \text{for } x > a.
\end{cases}$$

(a) Determine the probability current of the incident, the reflected and the transmitted part of the wavefunction.

(b) What is the reflection factor and transmission factor for $0 < E < V_0$ and for $E > V_0$?

$$R := \left| \frac{j_{\text{refl}}}{j_0} \right|, \quad T := \left| \frac{j_{\text{trans}}}{j_0} \right| ?$$

(c) Determine $T$ for $E = V_0$?

Problem 18. Baker-Hausdorff-Theorem

(a) Show that for arbitrary operators $\hat{A}$ and $\hat{B}$ and a complex number $x$ hold:

$$e^{xA}\hat{B}e^{-xA} = \hat{B} + x\left[ \hat{A}, \hat{B} \right] + \frac{1}{2!} x^2 \left[ \left[ \hat{A}, \hat{A} \right], \hat{B} \right] + \ldots. \quad (1)$$

Hint: Consider the taylor series of the function $\hat{f}(x) = e^{xA}\hat{B}e^{-xA}$

(b) Use the equation (1) to proof the Baker-Hausdorff-Theorem

$$e^{\hat{A}\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}\left[ \hat{A}, \hat{B} \right]} = e^{\hat{A}}e^{\frac{1}{2}\left[ \hat{A}, \hat{B} \right]}e^{\hat{B}}, \quad (2)$$

if $\left[ \hat{A}, \left[ \hat{A}, \hat{B} \right] \right] = \left[ \hat{B}, \left[ \hat{A}, \hat{B} \right] \right] = 0.$

Hint: Consider the function $\hat{f}(x) = e^{xA}e^{xB}$ and find a differential equation of first order by differentiation of $\hat{f}$ and use of $e^{-xA}e^{xA} = 1.$
Problem 19. Double-Delta-Potential
Consider two identical attractive delta potentials at distance $d$:

$$V(x) = -F \left( \delta(x - \frac{d}{2}) + \delta(x + \frac{d}{2}) \right),$$

where $F > 0$. Solve the stationary Schrödinger equation in the three intervals $-\infty < x \leq -d/2$, $-d/2 \leq x \leq d/2$, $d/2 \leq x < \infty$. Find all bound states and the corresponding energies.