Note: Please insert your assignment into the mailboxes 5th floor, building 46. The problems marked by a ☗ have to be submitted.

☗ **Problem 44.** *Zeeman effect - perturbation theory*
The magnetic moment of an electron in a central potential consists of orbital angular momentum and spin:

$$\hat{\vec{\mu}} = \mu_B(\hat{\vec{L}} + 2\hat{\vec{S}})$$

where $\mu_B := e\hbar/2m$ is the Bohr magneton. The energy of the magnetic moment in a magnetic field $\vec{B}$ is given by

$$\hat{H}_Z = -\hat{\vec{\mu}} \cdot \vec{B}.$$

For small magnetic field strength, due to $\hat{H}_Z$ the degenerate states of the system become split states with fixed l and j. Determine the splitting in a constant, homogeneous magnetic field $\vec{B} = (0, 0, B)$ for $l = 1$ and $j = l - s = 1/2$ in first order perturbation theory.

☗ **Problem 45.** *Modification of the Coulomb potential - pertubation Theory*
Considering quantum electrodynamical corrections the Coulomb potential at small distances is modified

$$\frac{1}{r} \to \frac{1}{r}\left[1 + \frac{\alpha}{3\pi}\int_0^1 \frac{dy}{y}\left(1 + \frac{1}{2}y\right)\sqrt{1-y}\ \exp\left\{-\frac{r/a_e}{\alpha\sqrt{y}}\right\}\right]$$

where $\alpha = 1/137.036...$ is the fine structure constant and a_e is the Bohr radius. Determine the shift of the hydrogen atom ground state in first order perturbation theory. The electron spin induced degeneracy of the states should be neglected.
Hint: Use the approximation of the integral

$$\int_0^1 dy \left(1 + \frac{1}{2}y\right)\sqrt{1-y} = \frac{4}{5}$$

for $\alpha \ll 1$.

Problem 46. *Anharmonic Oscillator*
Consider a one dimensional harmonic oscillator with a small anharmonic pertubation $\hat{H}_1$

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{m}{2}\omega_0^2\hat{x}^2$$

$$\hat{H}_1 = \alpha\sqrt{2}\frac{\hat{x}^3}{l_0^3}$$

where $l_0 = \sqrt{\frac{\hbar}{m\omega_0}}$ is the oscillator length and α has a positive value.
Determine the energy correction of the ground state of $\hat{H}_0$ due to perturbation $\hat{H}_1$ in the lowest *non-vanishing* order perturbation theory.
Hint: A product of k creation operators $\hat{a}^\dagger$ and l annihilation operators $\hat{a}$ can be applied in arbitrary order onto an eigenstate $|n\rangle$ of $\hat{a}^\dagger\hat{a}$ yielding a state proportional to $|n + k - l\rangle$.