Note: Please insert your assignment into the mailboxes 5th floor, building 46. The problems marked by a ☗ have to be submitted.

☗ **Problem 42.** *Spin precession*

Consider a spin-1/2 particle in an external magnetic field $\vec{B}(t) = (0, 0, B(t))$. Neglecting the motion of a particle the Hamiltonian is

$$\hat{H} = \frac{2\mu}{\hbar}\vec{B}(t)\cdot\hat{\vec{S}}. \tag{1}$$

At time $t = 0$ the state of the particle is given by the two-component vector

$$\chi(0) = \alpha\chi_+ + \beta\chi_-, \tag{2}$$

where $\chi_\pm$ are the eigenstates of $\hat{S}_z$ with eigenvalues $\pm\hbar/2$. Calculate the expectation values of $\langle\hat{S}_x(t)\rangle$, $\langle\hat{S}_y(t)\rangle$ and $\langle\hat{S}_z(t)\rangle$.

☗ **Problem 43.** *magnetic resonance*

Consider again a spin-1/2 particle in an external magnetic field as in problem 42. Now consider $B(t) = (-B_\perp\cos(\omega t), B_\perp\sin(\omega t), B_\parallel)$. The state of the particles at time t can be expressed by

$$\chi(t) = \alpha(t)\chi_+ + \beta(t)\chi_-. \tag{3}$$

(a) Show that the Schrödinger equation for $\chi(t)$ with the Hamiltonian $\hat{H}$ from Eq. (1) can be expressed as

$$\frac{d}{dt}\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = -i\begin{pmatrix}\Omega_\parallel & \Omega_\perp e^{i\omega t}\\ \Omega_\perp e^{-i\omega t} & -\Omega_\parallel\end{pmatrix}\cdot\begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix}, \tag{4}$$

with $\hbar\Omega_\parallel = \mu B_\parallel$ and $\hbar\Omega_\perp = \mu B_\perp$.

(b) Solve Eq. (4) with the initial conditions $\alpha(0) = 1$,$\beta(0) = 0$.

(c) At which frequency ω and at which time is $\chi(t) \propto \chi_-$?