1. **Problem 42. Spin precession**

Consider a spin-1/2 particle in an external magnetic field $\vec{B}(t) = (0, 0, B(t))$. Neglecting the motion of a particle the Hamiltonian is

$$\hat{H} = \frac{2\mu}{\hbar} \vec{B}(t) \cdot \hat{\vec{S}}. \quad (1)$$

At time $t = 0$ the state of the particle is given by the two-component vector

$$\chi(0) = \alpha \chi_\pm + \beta \chi_-,$$  

where $\chi_\pm$ are the eigenstates of $\hat{S}_z$ with eigenvalues $\pm \hbar/2$. Calculate the expectation values of $\langle \hat{S}_x(t) \rangle$, $\langle \hat{S}_y(t) \rangle$ and $\langle \hat{S}_z(t) \rangle$.

2. **Problem 43. Magnetic resonance**

Consider again a spin-1/2 particle in an external magnetic field as in problem 42. Now consider $B(t) = (-B_\perp \cos(\omega t), B_\perp \sin(\omega t), B_\parallel)$. The state of the particles at time $t$ can be expressed by

$$\chi(t) = \alpha(t) \chi_\pm + \beta(t) \chi_-.$$  

(a) Show that the Schrödinger equation for $\chi(t)$ with the Hamiltonian $\hat{H}$ from Eq. (1) can be expressed as

$$\frac{d}{dt} \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} = -i \begin{pmatrix} \Omega_\parallel & \Omega_\perp e^{i\omega t} \\ \Omega_\perp e^{-i\omega t} & -\Omega_\parallel \end{pmatrix} \cdot \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix}, \quad (4)$$

with $\hbar \Omega_\parallel = \mu B_\parallel$ and $\hbar \Omega_\perp = \mu B_\perp$.

(b) Solve Eq. (4) with the initial conditions $\alpha(0) = 1, \beta(0) = 0$.

(c) At which frequency $\omega$ and at which time is $\chi(t) \propto \chi_-$?