Note: Please upload your assignment in the olat course under "Übungsaufgaben". Only the problems marked by a ☗ have to be submitted.

☗ **Problem 12.** *Scattering from a box potential (10 points)*
Consider a particle with mass m moving along the x-axis in a potential

$$V(x) = \begin{cases} 0 & \text{for } |x| > a \\ V_0 & \text{for } |x| \leq a, \end{cases}$$

where $V_0 > 0$. A particle with energy $E > 0$ moves from $x = -\infty$ until it reaches the barrier. It has the wavefunction

$$\phi(x) = \begin{cases} e^{ikx} + re^{-ikx} & \text{for } x < -a \\ \beta_+ e^{Kx} + \beta_- e^{-Kx} & \text{for } -a \leq x \leq a \\ te^{ikx} & \text{for } x > a. \end{cases}$$

Hint: Depending on the energy, K is real or imaginary with $K = i\,k'$.

(a) Determine the probability current of the incident, the reflected and the transmitted part of the wavefunction.

(b) What is the reflection factor and transmission factor for $0 < E < V_0$ and for $E > V_0$

$$R := \left|\frac{j_{\text{refl}}}{j_0}\right|, \quad T := \left|\frac{j_{\text{trans}}}{j_0}\right| \ ?$$

(c) Determine T for $E = V_0$?

☗ **Problem 13.** *Baker-Hausdorff-Theorem (8 points)*

(a) Show that any arbitrary operators $\hat{A}$ and $\hat{B}$ and for a complex number x hold:

$$e^{x\hat{A}}\hat{B}e^{-x\hat{A}} = \hat{B} + \left[\hat{A},\hat{B}\right]x + \frac{1}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]x^2 + \dots. \tag{1}$$

Hint: Consider the taylor series of the function $\hat{f}(x) = e^{x\hat{A}}\hat{B}e^{-x\hat{A}}$

(b) Use the equation (1) to proof the Baker-Hausdorff-Theorem

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}[\hat{A},\hat{B}]} = e^{\hat{A}+\hat{B}}e^{\frac{1}{2}[\hat{A},\hat{B}]}, \tag{2}$$

if $\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] = \left[\hat{B},\left[\hat{A},\hat{B}\right]\right] = 0$.

Hint: Consider the function $\hat{f}(x) = e^{x\hat{A}}e^{x\hat{B}}$ and find a differential equation of first order by derivating $\hat{f}$ and the use of $e^{-x\hat{A}}e^{x\hat{A}} = \mathbb{1}$.

Problem 14. *Double-Delta-Potential*
Consider two identical attractive delta potentials at distance d:

$$V(x) = -F\left(\delta(x - \frac{d}{2}) + \delta(x + \frac{d}{2})\right),$$

where $F > 0$. Solve the stationary Schrödinger equation in the three subspaces $-\infty < x \leq -d/2$, $-d/2 \leq x \leq d/2$, $d/2 \leq x < \infty$. Find all bound states and the corresponding energies.