Note: Please upload your assignment in the olat course under "Übungsaufgaben". Only the problems marked by a ☗ have to be submitted.

☗ **Problem 39.** *Ritz method of variations* (6 points)
Consider the double-delta potential from problem 14. Determine the energies of the bound states using the Ritz method of variation. Use the ansatz

$$\phi(x) = \alpha_1 \phi_0(x - d/2) + \alpha_2 \phi_0(x + d/2)$$

($\alpha_1, \alpha_2 \in \mathbb{C}$) where ϕ_0 is the stationary wave function for a single delta-potential.

☗ **Problem 40.** *Greenberger-Horne-Zeilinger state* (8 points)
Given a state of 3 indistinguishable spin-1/2 particles

$$|\psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_{\mathrm{A}} |\uparrow\rangle_{\mathrm{B}} |\uparrow\rangle_{\mathrm{C}} - |\downarrow\rangle_{\mathrm{A}} |\downarrow\rangle_{\mathrm{B}} |\downarrow\rangle_{\mathrm{C}} \right)$$

where $|\uparrow\rangle_{\mathrm{A}}$ is the eigenstate of $\hat{\sigma}_z^{\mathrm{A}}$ of the eigenvalue $+1$ and $|\downarrow\rangle_{\mathrm{A}}$ is the eigenstate of the eigenvalue -1 and analogously for B and C. Show that $|\psi\rangle_{\mathrm{GHZ}}$ is an eigenstate of the following observables

$$\hat{\sigma}_y^{\mathrm{A}} \hat{\sigma}_y^{\mathrm{B}} \hat{\sigma}_x^{\mathrm{C}}, \quad \hat{\sigma}_y^{\mathrm{A}} \hat{\sigma}_x^{\mathrm{B}} \hat{\sigma}_y^{\mathrm{C}}, \quad \hat{\sigma}_x^{\mathrm{A}} \hat{\sigma}_y^{\mathrm{B}} \hat{\sigma}_y^{\mathrm{C}}, \quad \hat{\sigma}_x^{\mathrm{A}} \hat{\sigma}_x^{\mathrm{B}} \hat{\sigma}_x^{\mathrm{C}}.$$

What are the corresponding eigenvalues? Discuss the non-classical behaviour of $|\psi\rangle_{\mathrm{GHZ}}$. What result would you expect when measuring $\hat{\sigma}_x^{\mathrm{A}} \hat{\sigma}_x^{\mathrm{B}} \hat{\sigma}_x^{\mathrm{C}}$ *classically*. Meaning you assume classical random variables for the spins and a previous measurement of $\hat{\sigma}_y^{\mathrm{A}} \hat{\sigma}_y^{\mathrm{B}} \hat{\sigma}_x^{\mathrm{C}}$, $\hat{\sigma}_y^{\mathrm{A}} \hat{\sigma}_x^{\mathrm{B}} \hat{\sigma}_y^{\mathrm{C}}$ and $\hat{\sigma}_x^{\mathrm{A}} \hat{\sigma}_y^{\mathrm{B}} \hat{\sigma}_y^{\mathrm{C}}$ to yield the quantum mechanical result from above.

☗ **Problem 41.** *Two spin-1 particles* (6 points)
A system of two *distinguishable* spin-1 particles (without orbital angular momentum) possesses the eigenvalues $S = 0, 1, 2$ of the squared total spin $\hat{S}^2 = \hat{\vec{S}} \cdot \hat{\vec{S}}$ with $\hat{\vec{S}} = \hat{\vec{S}}_1 + \hat{\vec{S}}_2$. What are the limitations for *indistinguishable* spin-1 particles? Construct the common eigenstates of $\hat{S}^2$ and $\hat{S}_z$ of the particle pair in $\mathcal{H} \overset{sym}{\otimes} \mathcal{H}$ of the eigenstates $\{|-1\rangle, |0\rangle, |1\rangle\}$ of the considered observable for one particle.

Problem 42. *Two spin-$\frac{1}{2}$ particles in a box potential*
Consider two spin-$\frac{1}{2}$ particles move in a one-dimensional box with infinite high walls

$$V(x) = \begin{cases} 0 & 0 \leq x \leq L \\ \infty & x < 0, x > L. \end{cases} \tag{1}$$

(a) What is the ground state energy of the spin triplet state?

(b) What is the ground state energy in the spin-singlet state?

(c) Consider in lowest order perturbation theory an interaction

$$V_1(x_1, x_2) = -\lambda\delta(x_1 - x_2) \qquad \text{with } \lambda > 0. \tag{2}$$

Which effect has the perturbation on the ground state energy in case (a) and which in case (b)?