"*Anyone who is not shocked by quantum theory has not understood it.*" – Niels Bohr

Problem 1. *Double Slit Experiment*
Consider the sketched double slit experiment. If only one slit is opened, the wave amplitude on the screen is

$$\psi_1(y) = \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} e^{i(wt-ay)} \tag{1}$$

and

$$\psi_2(y) = \frac{1}{\sqrt{2}} e^{-\frac{y^2}{2}} e^{i(wt-(a+b)y)}, \tag{2}$$

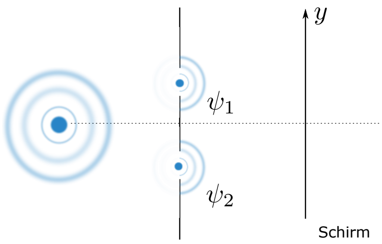
if the other slit is opened. Compute the total intensity I on the screen if:

(a) The intensities are summed as

$$I = |\psi_1(y)|^2 + |\psi_2(y)|^2. \tag{3}$$

(b) The wave functions are summed as

$$I = |\psi_1(y) + \psi_2(y)|^2. \tag{4}$$



Problem 2. *Wave Functions*
Let the wave function of a particle in one dimension be

(a) $\psi_1 = A_1 e^{-\frac{x^2}{4}}$
(b) $\psi_2 = A_2 x e^{-\frac{x^2}{8}}$
(c) $\psi_3 = A_3 \left(e^{-\frac{x^2}{4}} + x e^{-\frac{x^2}{8}} \right)$

Determine the absolute value of A_1, A_2 and A_3. Note that wave functions are normalized

$$\int_{-\infty}^{\infty} \mathrm{d}x |\psi_j|^2 = 1. \tag{5}$$

Problem 3. *Operators*
The following operators are given:

$$
\begin{aligned}
&(i) \quad \hat{D} = \frac{\partial}{\partial x} \ \text{ with } \ \hat{D}\psi(x) = \frac{\partial \psi(x)}{\partial x} \\
&(ii) \quad \hat{\mathbb{1}} \ \text{ with } \ \hat{\mathbb{1}}\psi(x) = \psi(x) \\
&(iii) \quad \hat{F} \ \text{ with } \ \hat{F}\psi(x) = f(x)\psi(x) \\
&(iv) \quad \hat{P} \ \text{ with } \ \hat{P}\psi(x) = \psi(x)^3 + 3\psi(x)^2 - 4 \\
&(v) \quad \hat{Q} \ \text{ with } \ \hat{Q}\psi(x) = \int_0^1 \mathrm{d}x \psi(x).
\end{aligned}
$$

(a) which of these operators are linear?

(b) $\hat{A}^{-1}$ with $\hat{A} = \hat{D}, \hat{\mathbb{1}}, \hat{F}$ is the inverse of A iff the inverse of A exist. Construct the inverse of A.

(c) Show that $i\hat{D}$ is self-adjoint.

Problem 4. *Adjoint Operators*
Show:

(a) $\left(\hat{A}\hat{B}\right)^{\dagger} = \hat{B}^{\dagger}\hat{A}^{\dagger}$

(b) $(\hat{A}^{\dagger})^{\dagger} = \hat{A}$

(c) If an operator $\hat{B}$ has the eigenvalue b with $b \neq b^*$, then $\hat{B} \neq \hat{B}^{\dagger}$.

The *commutator* of two operators $\hat{A}, \hat{B}$ is defined as

$$\left[\hat{A}, \hat{B}\right] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}. \tag{6}$$

(d) Show: If $\hat{A} = \hat{A}^{\dagger}$ and $\hat{B} = \hat{B}^{\dagger}$ then

$$\left[\hat{A}, \hat{B}\right]^{\dagger} = -\left[\hat{A}, \hat{B}\right] \tag{7}$$

Problem 5. *Complex Conjugation*
Consider an operator $\hat{C}$ with the following property

$$\hat{C}\psi(x) = \psi(x)^* \tag{8}$$

(a) Is $\hat{C}$ hermitian?

(b) What are the eigenfunctions of $\hat{C}$?

(c) What are the eigenvalues of $\hat{C}$?

Consult "Introductory Quantum Mechanics" by Richard Liboff chapter 2 and chapter 4.