Problem 1. – *Sigma-matrices*
Calculate all 6 matrices $\sigma_{\mu\nu}$, that fulfil

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right],$$

where $\gamma_0 = \beta$ and $\gamma^j = \beta\alpha_j$.

Problem 2. – *Lorentz-transformation*
Show, that the transformation-operator for Lorentz-transformations of Dirac-spinors

$$S = e^{-\frac{i}{\hbar}\omega\sigma_{\mu\nu}I^{\mu\nu}}$$

fulfils

$$S^{-1} = \gamma_0 S^\dagger \gamma_0.$$

Problem 3. – *Boost of a Dirac particle*
The following equation is an eigenstate of the Dirac-Hamiltonian for vanishing momentum $\overrightarrow{p} = 0$:

$$\psi_1\left(\overrightarrow{r}, t\right) = \frac{1}{\sqrt{V}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} e^{-\frac{i}{\hbar}m_0c^2t}$$

Start with ψ_1 and construct a solution for finite momentum $\overrightarrow{p} = (p_x, 0, 0)$ by using a Lorentz-transformation and compare your result with the one from **Problem sheet 7, prob.3**.

Problem 4. – *Invariance under gauge transformation*
Show explicitly, that the Dirac-equation in an external electromagnetic field

$$i\hbar\frac{\partial}{\partial t}\psi = \left\{c\overrightarrow{\alpha}\left(\overrightarrow{p} - \frac{e}{c}\overrightarrow{A}\right) - eA_0 + m_0c^2\right\}\psi$$

is invariant under gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \frac{\partial\lambda(x)}{\partial x^\mu}$$

$$\psi(x) \to \psi'(x) = \psi(x)\, e^{-\frac{i\lambda(x)}{\hbar c}}$$