Problem 1. – *Many particle operators*
Show, that the momentum operator $\widehat{\vec{p}} = -i\hbar\vec{\nabla}$ is a single particle operator, but the position operator $\widehat{\vec{x}} = \vec{x}$ isn't. Use the states from problem sheet 6 (problem 2):

$$\Psi_{(+)} = \int d^3p A(p) \begin{pmatrix} m_0c^2 + \hbar\omega_p \\ m_0c^2 - \hbar\omega_p \end{pmatrix} e^{i\left(\frac{\vec{p}\cdot\vec{x}}{\hbar} - \omega_p t\right)},$$

$$\Psi_{(-)} = \int d^3p A(p) \begin{pmatrix} m_0c^2 - \hbar\omega_p \\ m_0c^2 + \hbar\omega_p \end{pmatrix} e^{i\left(\frac{\vec{p}\cdot\vec{x}}{\hbar} + \omega_p t\right)},$$

where $\hbar\omega_p = c\sqrt{p^2 + m_0^2c^2}$.

Problem 2. – *Useful identity*
Proof the following statement:

$$\left(\vec{\sigma}\cdot\vec{A}\right)\left(\vec{\sigma}\cdot\vec{B}\right) = \left(\vec{A}\cdot\vec{B}\right)\mathbb{1}_2 + i\vec{\sigma}\cdot\left(\vec{A}\times\vec{B}\right)$$

Problem 3. – *Free Dirac equation*
Find the stationary solution of the free Dirac equation:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left(c\vec{\alpha}\cdot\vec{p} + \beta m_0c^2\right)\Psi.$$

Use the ansatz: $\Psi = \begin{pmatrix} \phi_0 \\ \chi_0 \end{pmatrix} e^{\frac{i}{\hbar}\left(\frac{\vec{p}\cdot\vec{r}}{\hbar} - Et\right)}$.
What is the non-relativistic limit?

Problem 4. – *Velocity operator*
Find the true single-particle-velocity operator of a Dirac-particle $\left[\frac{d\widehat{\vec{x}}}{dt}\right]$ and compare the result with

$$\frac{d\widehat{\vec{x}}}{dt} = \frac{-i}{\hbar}\left[\widehat{\vec{x}}, H\right] = c\vec{\alpha}.$$

What are the eigenvalues and eigenstates of the true single-particle-velocity operator? Compare the results with a classical relativistic particle.